J.Partial Diff. Eqs. 12(1999), 55-62
©International Academic Publishers Vol.12, No.1

APPROXIMATE INERTIAL MANIFOLDS OF STRONGLY DAMPED NONLINEAR WAVE EQUATIONS *

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(Received Dec. 17, 1996)

Abstract In this paper we consider a class of strongly damped nonlinear wave equations. By the transformation of unknown functions and decomposition of operators, we construct a family of approximate inertial manifolds, and obtain the estimate of orders of approximation of such manifolds to solution orbits.

Key Words Approximate inertial manifolds; strong damping; nonlinear wave equation.

Classification 35B40, 35P10.

1. Introduction

In this paper, we will study the approximate inertial manifolds of strongly damped nonlinear wave equations with initial boundary value:

$$u_{tt} = \alpha u_{xxt} + \sigma(u_x)_x - f(u) + g(x), \quad x \in (0,1), \quad t \in [0, +\infty) \tag{1.1}$$

$$u(0) = u_0, \quad u_t(0) = u_1 \tag{1.2}$$

$$u(0,t) = u(1,t) = 0 \tag{1.3}$$

where $\sigma(s)$ is a smooth function with the following property

$$\sigma(0) = 0, \quad \sigma'(s) \geq \gamma_0 > 0, \quad \forall s \in \mathrm{R} \tag{1.4}$$

where α, γ_0 are positive constants. As for nonlinear item $f(u)$, we assume that f is smooth and satisfies the following conditions:

(i) $$\lim_{|s|\to\infty} \frac{F(s)}{|s|^2} \geq 0; \tag{1.5}$$

(ii) there exists a positive constant ω such that

* The project supported by the Applied Basic Research Foundation of Yunnan Province.

$$\lim_{|s|\to\infty} \frac{s\cdot f(s) - \omega \cdot F(s)}{|s|^2} \geq 0 \tag{1.6}$$

where $F(s) = \int_0^s f(s)ds$.

Let $I = (0,1)$, $L^2(I)$ be the usual Hilbert space of measurable functions which are square integrable on I, with the norm $|v|_0 = \left[\int_I |v|^2 dx\right]^{1/2}$, and inner product $(u,v) = \int_I uv dx$. Denote $A = -\partial_{xx}$, the Laplacian operator on $L^2(I)$, its domain is denoted by $D(A)$. Define $X = D(A) \times L^2(I)$, the norm of X will be denoted by $\|\cdot\|$, $\|(u,v)\| = (\|u\|_2^2 + |v|_0^2)^{1/2}$, where $\|u\|_2^2 = |Au.|_0^2$, $|\cdot|_0$ is the norm of $L^2(I)$.

The problem (1.1)–(1.3) arises when one considers the purely longitudinal motion of a homogeneous bar. This problem is studied in lot of literature. When f and g vanish, the existence and stability of classical solutions were studied by [1], [2]. The existence of solutions $(u,u_t) \in W^{1,\infty} \times W^{1,2}$ was proved by [3].

When f and g do not vanish and $\sigma(s)$ is nonlinear, Berkaliev [4], [5] studied the (E_0, E) attractor and its structure. Recently, in [6] the authors obtained the global existence and uniqueness of solutions $(u,u_t) \in C(0,\infty,X)$ and proved the existence of global compact attractor and its finite dimensionality property. On the other hand, there were many results to the inertial manifolds and approximate inertial manifolds for the nonlinear evolution equations of parabolic type. (See [7] and its references). But for the nonlinear wave equations, it is yet a difficult problem. Recently, K.S. Chueshov [8] studied the approximate inertial manifolds of strongly dampled nonlinear wave equations

$$u_{tt} + \alpha u_t - \Delta u + f(u) = g(x) \tag{1.7}$$

Under some assumption of $f(u)$, the author constructed a family of approximate inertial manifolds and obtained the estimate of orders of approximation of such manifolds to solution orbits. In this paper, by means of the transformation of unknown functions and decomposition of operators, we construct a family of approximate inertial manifolds $M(t)$ for the problem (1.1)–(1.3), our main results are

Theorem 1 *Suppose* $\sigma(s)$ *and* $f(s)$ *satisfy* (1.4), (1.5) *and* (1.6) *respectively, and* $g \in H^1(I)$. *For any* $(u_0, u_1) \in X$, *there exist a family of approximate inertial manifolds* $M_t(p,\dot p)$ *such that*

$$\text{dist}\,(M_t(p,\dot p), v(t)) \leq c\lambda_{N+1}^{-1/2} + c_1 e^{-\beta t} \tag{1.8}$$

when t *is sufficiently large, where* $v(t) = (u(t), u_t(t))$, *and* λ_{N+1} *is the* $(N+1)$*th eigenvalue* $[(N+1)\pi]^2$, *especially* $\text{dist}\,(M_t(p,\dot p), A) \leq c\lambda_{N+1}^{1/2} + c_1 e^{-\beta t}$, *where* A *is the global attractor of* (1.1)–(1.3) *in* X, *constant* c *is independent of* N.

2. Preliminary Results

In [6], the authors showed the following results