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# ASYMPTOTIC BEHAVIOR FOR GLOBAL SMOOTH SOLUTION TO A ONE-DIMENSIONAL NONLINEAR THERMOVISCOELASTIC SYSTEM

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**Abstract** This paper is concerned with asymptotic behavior, as time tends to infinity, of globally defined smooth (large) solutions to the system in one-dimensional nonlinear thermoviscoelasticity. Our results show that the global smooth solution approaches to the solution in the $H^1$ norm to the corresponding stationary problem, as time tends to infinity.

**Key Words** Global solution; asymptotic behavior; *a priori* estimates.
**Classification** 35M10, 73C35, 73B30.

## 1. Introduction

This paper is concerned with asymptotic behavior, as time tends to infinity, of global smooth (large) solutions to the system in one-dimensional nonlinear thermoviscoelasticity. The referential (Lagrangian) form of the conservation laws of mass, momentum, and energy for a one-dimensional material with the reference density $\rho_0 = 1$ is

$$u_t - v_x = 0 \tag{1.1}$$

$$v_t - \sigma_x = 0 \tag{1.2}$$

$$\Big(e + \frac{v^2}{2}\Big)_t - (\sigma v)_x + q_x = 0 \tag{1.3}$$

and the second law of thermodynamics is expressed by the Clausius-Duhem inequality

$$\eta_t + \Big(\frac{q}{\theta}\Big)_x \geq 0 \tag{1.4}$$

Here subscripts indicate partial differentiations, $u, v, \sigma, e, q, \eta$ and $\theta$ denote the deformation gradient, velocity, stress, internal energy, heat flux, specific entropy and temperature, respectively. We consider the problem (1.1)–(1.3) in the region $\{0 \leq x \leq 1, t \geq 0\}$ under the initial conditions

$$u(x,0) = u_0(x), v(x,0) = v_0(x), \theta(x,0) = \theta_0(x) \quad \text{on} \quad [0,1] \tag{1.5}$$

and the boundary conditions of the form

$$\sigma(0,t)=\gamma v(0,t),\sigma(1,t)=-\gamma v(1,t),\theta(0,t)=\theta(1,t)=T_0 \tag{1.6}$$

where $\gamma=0$ or $\gamma=1$, and $T_0>0$ is the reference temperature. The boundary condition (1.6) with $\gamma=1$, boundary damping, represents that the endpoints of the interval $[0,1]$ are connected to some sort of dash pot.

For one-dimensional homogeneous, thermoviscoelastic materials, $e,\sigma,\eta$ and $q$ are given by the constitutive relations (See [1])

$$e=e(u,\theta),\quad \sigma=\sigma(u,\theta,\theta_x),\quad \eta=\eta(u,\theta),\quad q=q(u,\theta,\theta_x) \tag{1.7}$$

which in order to be consistent with (1.4), must satisfy

$$\sigma(u,\theta,0)=\Psi_u(u,\theta),\quad \eta(u,\theta)=-\Psi_\theta(u,\theta) \tag{1.8}$$

$$(\sigma(u,\theta,w)-\sigma(u,\theta,0))w\geq 0,\quad q(u,\theta,g)g\leq 0 \tag{1.9}$$

where $\Psi=e-\theta\eta$ is the Helmholtz free energy function.

Before stating our results, let us first recall the related results on nonlinear one-dimensional thermoviscoelasticity. For solid-like materials, Dafermos [1], Dafermos and Hsiao [2] considered the following boundary conditions (stress free and thermally insulated):

$$\sigma(0,t)=\sigma(1,t)=0,\quad q(0,t)=q(1,t)=0,\quad t\geq 0 \tag{1.10}$$

and established the existence of global smooth solutions to (1.1)–(1.3), (1.5) and (1.10) by applying the Leray-Schauder fixed point theorem. The techniques in [1] work only for the case where one end of the body is strees-free while the other is fixed. By the same method as in [1] with necessary modifications, Jiang [3] established the global existence of smooth solution to the problem (1.1)–(1.3) and (1.5)–(1.6) with constitutive relations

$$e=e(u,\theta),\sigma=-p(u,\theta)+\mu(u)v_x,\ q=-k(u,\theta)\theta_x \tag{1.11}$$

where the viscosity $\mu(u)$ satisfies

$$\mu(u)u\geq\mu_0>0,\quad 0<u<+\infty \tag{1.12}$$

for some constant $\mu_0$. It is well-known that the large-time behavior of the system (1.1)–(1.3) is of great interest since the pressure function $p(u,\theta)$ is not necessary monotone in $u$. Unfortunately, the problem has been open till now. Hsiao and Luo [4] first considered a kind of solid-like material with the following constitutive relations

$$e=C_0\theta,\ \sigma=-p(u,\theta)+\mu(u)v_x,p(u,\theta)=f(u)\theta,q=-k(u)\theta_x \tag{1.13}$$