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CRITICAL FUJITA EXPONENTS FOR NONLOCAL REACTION DIFFUSION SYSTEMS*

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Abstract In this paper, we prove the existence of critical Fujita exponents for a class of nonlocal reaction diffusion systems. And it is proved that the critical Fujita exponents belong to the blow up case.

Key Words Nonlocal reaction-diffusion systems; blow up; global solutions; critical Fujita exponents.
Classification 35K50, 35K60.

1. Introduction

In this paper, we consider the problem

$$\begin{cases} u_t = \Delta u + \left(\int_{R^N} v^{\sigma_1}(y,t)dy\right)^{\frac{p_1}{\sigma_1}} v^{r_1}, & x \in R^N,\ t>0 \\ v_t = \Delta v + \left(\int_{R^N} u^{\sigma_2}(y,t)dy\right)^{\frac{p_2}{\sigma_2}} u^{r_2}, & x \in R^N,\ t>0 \\ u(x,0)=u_0(x),\ v(x,0)=v_0(x), & x\in R^N \end{cases} \tag{1}$$

where $p_i \geq 0$, $\sigma_i, r_i \geq 1$, $p_i + r_i > 1$, $i = 1, 2$, $u_0(x) \geq 0$, $v_0(x) \geq 0$ and $u_0(x), v_0(x) \in L^{\sigma_1}(R^N) \cap L^{\sigma_2}(R^N) \cap L^\infty(R^N)$.

It is well known that all nontrivial nonnegative solutions of the following Cauchy problem

$$\begin{cases} u_t = \Delta u + u^p, & x \in R^N,\ t>0 \\ u(x,0)=u_0(x), & x\in R^N \end{cases} \tag{2}$$

blow up in finite time if $1 < p \leq p_c \equiv 1 + \dfrac{2}{N}$ while global solutions exist when $p > p_c$ and the initial data are small (See [1, 2], etc.). The number p_c is often called critical

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Fujita exponent (See [3]). This result has been extended in a number of directs (See [4, 5]).

In this paper, we shall extend the Fujita type results to the nonlocal problem (1). Throughout this paper, we let (α, β) solve the systems

$$\begin{cases} \alpha - (p_1 + r_1)\beta = -1 - \dfrac{Np_1}{2\sigma_1} \\ -(p_2 + r_2)\alpha + \beta = -1 - \dfrac{Np_2}{2\sigma_2} \end{cases} \tag{3}$$

i.e.

$$\alpha = \frac{(1 + \frac{Np_1}{2\sigma_1}) + (1 + \frac{Np_2}{2\sigma_2})(p_1 + r_1)}{(p_1 + r_1)(p_2 + r_2) - 1}, \quad \beta = \frac{(1 + \frac{Np_2}{2\sigma_2}) + (1 + \frac{Np_1}{2\sigma_1})(p_2 + r_2)}{(p_1 + r_1)(p_2 + r_2) - 1}$$

Our main result reads as follows:

Theorem (i) *If* $\max\{\alpha, \beta\} \geq \frac{N}{2}$, *then all positive solutions of* (1) *blow up in finite time;*

(ii) *If* $\max\{\alpha, \beta\} < \frac{N}{2}$, *then the solutions of* (1) *below up in finite time when the initial values are large while global solutions exist when the initial values are small.*

Many physical phenomena were formulated into nonlocal mathematical models (See [6–9] and references therein) and studied by many authors. Bebernes and Bressan [7] studied an ignition model for a compressible reactive gas which is a nonlocal reaction diffusion equation. Pao [8] also discussed a nonlocal reaction diffusion equation arising from combustion theory.

This paper is organized as follows. In Section 2, we state the local existence and uniqueness of solutions, and establish the comparison principle. In Section 3, we will prove our main Theorem.

2. Local Existence and Uniqueness, Comparison Principle

We first prove

Proposition 1 (Local Existence and Uniqueness) (1) *has a unique local solution.*

Proof We first prove the existence. By assumptions, there exists $\phi(x) \in L^{\sigma_1}(R^N) \cap L^{\sigma_2}(R^N) \cap L^\infty(R^N)$ such that $u_0, v_0 \leq \phi(x)$ for $x \in R^N$.

Let $(c_1(t), c_2(t))$ be the solution of the ordinary differential system

$$\begin{cases} c_1'(t) = \|\phi\|_{\sigma_1}^{p_1} \|\phi\|_\infty^{r_1 - 1} c_2^{p_1 + r_1}(t), & t > 0 \\ c_2'(t) = \|\phi\|_{\sigma_2}^{p_2} \|\phi\|_\infty^{r_2 - 1} c_1^{p_2 + r_2}(t), & t > 0 \\ c_1(0) = c_2(0) = 1 \end{cases} \tag{4}$$

Denote the maximal existence interval of the solution to (4) by $[0, 2\varepsilon)$. Set

$$E = \{(u(t), v(t)) \mid u(t), v(t) : [0, \varepsilon] \to L^{\sigma_1}(R^N) \cap L^{\sigma_2}(R^N) \cap L^\infty(R^N)$$