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DIFFERENCE METHOD OF GENERAL SCHEMES WITH INTRINSIC PARALLELISM FOR ONE-DIMENSIONAL QUASILINEAR PARABOLIC SYSTEMS WITH BOUNDED MEASURABLE COEFFICIENTS*

Zhou Yulin and Yuan Guangwei
(Laboratory of Computational Physics, Center of Nonlinear Studies, Institute of Applied Physics and Computational Mathematics, P.O. Box 8009, Beijing 100088, China)

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Abstract The difference method of the general finite difference schemes with intrinsic parallelism for the boundary value problem of the quasilinear parabolic system is studied without assuming heuristically that the original boundary value problem has the unique smooth vector solution.

Key Words Difference scheme; intrinsic parallelism; quasilinear parabolic system; convergence; stability.

Classification 35K55, 65M12.

1. Introduction

1. In [1] the finite difference methods with intrinsic parallelism for the boundary value problems of the quasilinear parabolic system are studied, where the difference approximations for the derivatives of second order are taken to be the various linear combinations of the two kinds of difference quotients. In [2–4] the general finite difference schemes having the intrinsic character of parallelism for the boundary value problems of the nonlinear parabolic system are discussed under the assumption that there is a unique smooth solution for the original problem.

In this paper we at first construct a general finite difference scheme with intrinsic parallelism for the boundary value problems of the quasilinear parabolic systems with bounded measurable coefficients. On each levels of grids and each grid points, the difference approximations for the derivatives of second order are taken to be in general the various linear combinations of the four kinds of difference quotients: the scheme ahead $\mathcal{A} = \dfrac{u_{j+1}^n - u_j^n - u_j^{n+1} + u_{j-1}^{n+1}}{h^2}$, the backward scheme $\mathcal{B} = \dfrac{u_{j+1}^{n+1} - u_j^{n+1} - u_j^n + u_{j-1}^n}{h^2}$,

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the scheme on the top cover of the grid $\mathcal{C} = \dfrac{u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}}{h^2}$, and the downward scheme $\mathcal{D} = \dfrac{u_{j+1}^{n} - 2u_j^{n} + u_{j-1}^{n}}{h^2}$. Then we prove the existence and uniqueness of the discrete vector solution of the general difference scheme with intrinsic parallelism and also the convergence behavior of the discrete vector solutions to the unique generalized vector solution of the original problem of the quasilinear parabolic systems. And also the stability of the general difference scheme with intrinsic parallelism is justified in the sense of the continuous dependence of the discrete vector solutions on the discrete values of data of the original problems for the quasilinear parabolic systems. It is noticed that in the present study the coefficients and the free term are allowed to be discontinuous, and we don't assume heuristically that the original boundary value problem for the quasilinear parabolic system has the unique smooth vector solution.

2. Difference Schemes with Intrinsic Parallelism

2. Let us now consider the boundary value problems for the quasilinear parabolic systems of second order of the form

$$u_t = A(x,t,u)u_{xx} + B(x,t,u)u_x + f(x,t,u) \tag{1}$$

where $u(x,t) = (u_1(x,t), \cdots, u_m(x,t))$ is the m-dimensional vector unknown function $(m \geq 1)$, $u_t = \dfrac{\partial u}{\partial t}$, $u_x = \dfrac{\partial u}{\partial x}$ and $u_{xx} = \dfrac{\partial^2 u}{\partial x^2}$ are the corresponding vector derivatives. The matrix $A(x,t,u)$ is $m \times m$ positive definite coefficient matrix, and $B(x,t,u)$ is the $m \times m$ matrix, and $f(x,t,u)$ is the m-dimensional vector function. Let us consider in the rectangular domain $Q_T = \{0 \leq x \leq l,\ 0 \leq t \leq T\}$ with $l > 0$ and $T > 0$, the problem for the systems (1) with the boundary value condition

$$u(0,t) = u(l,t) = 0 \tag{2}$$

and the initial value condition

$$u(x,0) = \varphi(x) \tag{3}$$

where $\varphi(x)$ is a given m-dimensional vector function of variable $x \in [0,l]$.

Suppose that the following conditions are fulfilled.

(I) For any fixed $u \in R^m$, $A(x,t,u)$, $B(x,t,u)$ and $f(x,t,u)$ are bounded measurable functions with respect to $(x,t) \in Q_T$; for any fixed $(x,t) \in Q_T$, $A(x,t,u)$, $B(x,t,u)$ and $f(x,t,u)$ are continuous with respect to $u \in R^m$; and $|A(x,t,u)| \leq A_0(|u|)$, where $A_0(s)$ is a monotone function; and there are constants $B_0 > 0$, $C > 0$ such that $|B(x,t,u)| \leq B_0$, $|f(x,t,u)| \leq |f(x,t,0)| + C|u|$.

(II) There is a constant $\sigma_0 > 0$, such that, for any vector $\xi \in R^m$, and for $(x,t) \in Q_T$ and $u \in R^m$,

$$(\xi, A(x,t,u)\xi) \geq \sigma_0 |\xi|^2$$