J.Partial Diff. Eqs. 13(2000), 163–175
©International Academic Publishers Vol.13, No.2

THE LONG TIME BEHAVIORS OF NON-AUTONOMOUS NAVIER-STOKES EQUATIONS WITH LINEAR DAMPNESS ON THE WHOLE R^2 SPACE

Zhao Chunshan and Li Kaitai
(School of Science, Xi'an Jiaotong University, Xi'an 710049, China)
(Received Jan. 7, 1999; revised Jun. 7, 1999)

Abstract In this paper, the long time behaviors of non-autonomous Navier-Stokes equations with linear dampness on the whole R^2 space are considered. The existence of uniform attractor is proved when the external force terms satisfy suitable conditions. Moreover, the upper bounds of the uniform attractor's Hausdorff and Fractal dimensions are obtained.
Key Words Non-autonomous Navier-Stokes equations; linear dampness; uniform attractor; Hausdorff and Fractal dimension.
1991 MR Subject Classification 35B40, 35Q10, 76E10.
Chinese Library Classification O175.29, O175.26.

1. Introduction

As fundamental equations in fluid dynamics, Navier-Stokes equations have been studied extensively. Some results of their long time behaviors have been obtained when external force terms are independent of time (See [1–6]). If external forces are independent of time, the operator $S(t) : u_0 \to u(t)$ satisfies the semigroup properties and the long time behaviors of Navier-Stokes equations on two-dimensional bounded domains or strip-like unbounded domains can be described by global attractors. But if the external forces are dependent on time, the operator $S(t)$ defined above does not satisfy the semigroup properties anymore. So how to describe the long time behaviors of non-autonomous Navier-Stokes equations becomes very important. Since the definition of uniform attractor was given in [7], most researchers agree that the long time behaviors of non-autonomous dissipative evolution systems can be described by uniform attractors. The long time behaviors of non-autonomous Navier-Stokes on two-dimensional bounded domains have been studied in the reference [8]. Our work is motivated by the above works and we consider the following non-autonomous Navier-Stokes equations with linear dampness on the whole R^2 space.

$$\frac{\partial u}{\partial t} - \mu\Delta u + \alpha u + (u\cdot\nabla)u + \nabla p = f(x,t) \tag{1.1}$$

$$\operatorname{div} u = 0 \tag{1.2}$$

$$\lim_{|x|\to\infty} u = 0 \tag{1.3}$$

$$u(x,\tau) = u_\tau(x) \tag{1.4}$$

In the above equations, $\mu > 0$ is the viscosity number of fluids, $\alpha > 0$ is dampness constant. It is obvious that (1.1) is Navier-Stokes equations when $\alpha = 0$, $u = (u_1, u_2)^T$ is velocity vector, p is the pressure term, $f(x,t)$ is external force term depending on time t, $\tau \in R$ is any initial time, $u_\tau(x)$ is a initial velocity vector.

2. Preliminaries

At first, let us introduce some functional spaces as follows.

$$H = \{u : u \in (L^2(R^2))^2, \quad \operatorname{div} u = 0\}$$

$(\cdot,\cdot)$, $|\cdot|_2$ denote the inner product and norm in H respectively.

$$V = \{u : u \in (H^1(R^2))^2, \ \operatorname{div} u = 0\}$$

$\|\cdot\|_1$ dedotes the norm in V, V' is the dual space of V. $\langle\cdot,\cdot\rangle$ denotes the dual product of V and V'. $H^m = H \cap (H^m(R^2))^2$, $H^m(R^2)$ are the common Sobolev spaces. $\|\cdot\|_m$ denotes the norm in H^m. $P : (L^2(R^2))^2 \to H$ is the Leray projecting operator. $Au = -P\Delta u$ is the Stokes operator.

Denote:

$$B(u,v) = (u \cdot \nabla)v = \sum_{i=1}^{2} u_i \frac{\partial v_j}{\partial x_i}$$

$$b(u,v,w) = \langle B(u,v), w\rangle = \sum_{i,j=1}^{2} \int_{R^2} u_i \frac{\partial v_j}{\partial x_i} w_j dx$$

$$((u,v)) = (\nabla u, \nabla v), \quad \|u\|^2 = ((u,u)), \ \forall u,v \in V, \ D(A) = H \cap (H^2(R^2))^2$$

For any Banach space $E, BC(R,E)$ is the Banach space consisting of all bounded continuous functions from R to E with the norm $|\cdot|_{BC(R,E)} = \max_{t\in R} |\cdot|_E$, where $|\cdot|_E$ is the norm in E.

Let $T(h)$ be a translation operator along time-axis defined as follows:

$$T(h)f(x,t) = f(x,t+h)$$

Later on, we make the following assumptions (H_1):

(i) $f(x,t)$ is almost periodic or quasiperiodic in time t,

(ii) $f(x,t) \in BC(R,V)$, $\dfrac{\partial f}{\partial t} \in BC(R,H)$, $[\ln\ln(e^e + |x|)]^{1/2} f(x,t) \in BC(R,H)$.