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ABOUT THE EXISTENCE TIME OF SOLUTIONS FOR FIELD EQUATIONS IN ONE SPACE DIMENSION

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Abstract In this paper, we study the lower bounds problem for the existence time of solutions to the different massive Dirac-Klein-Gordon equations and with different massive Klein-Gordon equations, in one space dimension, for weakly decaying Cauchy data, of size ε. The results assert that the existence time is (almost) larger than ε^{-4}.
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1. Introduction

One of the important nonlinear interactions encountered in field theory is the following Dirac-Klein-Gordon equations coupled through a Yakawa interaction,

$$\begin{cases} -i\gamma^\mu \partial_\mu \psi + M\psi = \phi V \psi \\ \Box\phi + m^2\phi = ig_0\tilde{\psi}\gamma^0\gamma^5\psi + g_1\tilde{\psi}\gamma^0\psi \end{cases} \tag{1.1}$$

where V is a complex 4×4 matrix such that $\tilde{V}\gamma^0 = \gamma^0 V$, M and m are nonnegative real constants, and g_0 and g_1 are real constants. The Dirac matrices γ^μ, $\mu = 0, 1, 2, 3, 5$, are defined by

$$\gamma^0 = \begin{pmatrix} id & 0 \\ 0 & -id \end{pmatrix}, \quad \gamma^j = \begin{pmatrix} 0 & \sigma^j \\ \sigma^j & 0 \end{pmatrix}, \quad j = 1, 2, 3, \ \gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$$

such that

$$\gamma^5 = \begin{pmatrix} 0 & id \\ id & 0 \end{pmatrix}, \quad \text{and} \quad \gamma^0\gamma^5 = \begin{pmatrix} 0 & id \\ -id & 0 \end{pmatrix}$$

where

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \text{and} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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are the Pauli matrices. ψ is a complex 4-dimensional vectors (called spinors [1]), $\bar{\psi}$ denotes the conjugate transpose of ψ, and ϕ is a real scalar field. This system comes from physics, the fundamental example is the pseudoscalar Yukawa model of nuclear forces.

There exists a global solution for (1.1) with small, smooth initial data, decaying rapidly enough at infinity initial data in the following cases:

For three space dimensions case, if the mass is not zero ($M \neq 0$, $m \neq 0$), the system (1.1) is equivalent to a system of Klein-Gordon equations with quadratic nonlinearities studied by S. Klainerman [2]. The key point is the $L^\infty(\mathbf{R}^3_x)$ norm decay as $|t|^{-1-\varepsilon}$, $\varepsilon > 0$. For one space dimension case, it was studied by J. M. Chadam [3].

If the massive $M = m = 0$, the system (1.1) is comformal invarience and the existence of the global solution is established by Y. Choquet-Bruhat [4] (see [5] and [6] also).

If the massive $M \neq 0$, $m = 0$, the global Cauchy problem is well posed, proved by A. Bachelot [7], and it is also true if the nonlinearities satisfy some algebraic conditions related to the Lorentz invariance, the null condition and the compatibility of a sesquilinear form with the Dirac system.

If we use weakly decaying condition instead of rapidly decaying one in Cauchy data, the only result for (1.1) with special case $M = m = 1$ was established by the author in [8].

Another of important equations is Klein-Gordon equations.

For Klein-Gordon equations:

$$\begin{cases} \Box u_1 + m_1^2 u_1 = F_1(u, u', u'') \\ \Box u_2 + m_2^2 u_2 = F_2(u, u', u'') \end{cases} \tag{1.2}$$

where m_1 and m_2 are two massive constants, $u = (u_1(x,t), u_2(x,t))$ is a function in $\mathbf{R} \times \mathbf{R}^d$, u' (resp. u'') is the derivatives of Order 1 (resp. 2) of u with respect to their arguments, $F = (F_1, F_2)$ is a regular function vanishing of second order at O, and F is linear in u'', and $\Box$ is a d'Alembert operator. For this equation, when $d \geq 3$, Klainerman [9] and Shatah [10] have proved that there exists a global solution of the Cauchy problem to (1.2) for the above condition's initial data. For $d = 2$ (resp. 1), Hörmander, in his monograph [11], has proved that the Cauchy problem with data in C^∞, of size ε, admits a solution in $[-T_\varepsilon, T_\varepsilon]$ with $\liminf_{\varepsilon \to 0}(\varepsilon \log T_\varepsilon) = +\infty$ (resp. $\liminf_{\varepsilon \to 0} \varepsilon\sqrt{T_\varepsilon} = +\infty$.), and there is a conjecture for dimensional 2, the solution exists globally. The conjecture has been proved by Geogiev-Popivanov [12] for special nonlinearities, and then, Kosecki [13], Simon and Taflin [14], and Ozawa, Tsutaya and Tsutsumi [15] with null condition nonlinearities.

If the condition of initial data is replaced by weakly decaying, of size ε, the only result that can be found is obtained by Delort in [16] for one dimension, and in [17] for multidimension with periodic initial data. However, we mainly deal with the first problem here, for the second, one can extend the conclusion to the different massive