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THE DYNAMICS OF SINE-GORDON SYSTEM WITH DIRICHLET BOUNDARY CONDITION*

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Abstract We prove the existence of the global attractor of Sine-Gordon system with Dirichlet boundary condition and show the attractor is the unique steady state when the damping constant and the diffusion constant are sufficiently large.
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1. Introduction

In physics the Sine-Gordon equation is used to model the dynamics of a Josephon junction in superconductivity theory. For a single junction the governing equation is an ordinary equation similar to the pendulum equation. A coupled system of such equations appears when we consider a family of coupled junctions, and the continuous case is modeled by the Sine-Gordon equation or system. In [1] the existence of attractor and the estimate of dimension of Sine-Gordon equation are considered. In [2] we can find the derivation of Sine-Gordon system with Dirichlet boundary condition. In [3] the author proved the existence of the global attractor of Sine-Gordon system with Dirichlet boundary condition and gave the dimension estimate of it. In all the above equations the forcing terms are all independent of time. In this paper we consider the Sine-Gordon system with Dirichlet boundary condition when the forcing term is periodic in time. We will prove the existence of the global attractor and show the attractor is the unique periodic steady state when the damping constant and the diffusion constant are sufficiently large. When the forcing term is independent of time, [3] showed the system is a gradient system and the global attractor is the unique steady state when the damping constant and diffusion constant are sufficiently large. But in fact when the global attractor is the unique steady state, it would not dependent on the damping constant since the system for the steady state is not dependent on it. In this paper we just get rid of this limitation. On the other hand, since the system is a gradient system, it is of sense to study the steady states more sufficiently. We will do some essential

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work in this direction, not only prove the uniqueness of the steady state for wider range of parameters but also show the effect of the coupling constant on the uniformness of the steady state.

Consider

$$\begin{cases} \dfrac{\partial^2 u_1}{\partial t^2} + \gamma \dfrac{\partial u_1}{\partial t} - d\triangle u_1 + \beta \sin u_1 + K(u_1 - u_2) = f_1(x,t) \\ \dfrac{\partial^2 u_2}{\partial t^2} + \gamma \dfrac{\partial u_2}{\partial t} - d\triangle u_2 + \beta \sin u_2 + K(-u_1 + 2u_2 - u_3) = f_2(x,t) \\ \vdots \\ \dfrac{\partial^2 u_i}{\partial t^2} + \gamma \dfrac{\partial u_i}{\partial t} - d\triangle u_i + \beta \sin u_i + K(-u_{i-1} + 2u_i - u_{i+1}) = f_i(x,t) \\ \vdots \\ \dfrac{\partial^2 u_n}{\partial t^2} + \gamma \dfrac{\partial u_n}{\partial t} - d\triangle u_n + \beta \sin u_n + K(-u_{n-1} + u_n) = f_n(x,t) \\ u_i|_{\partial\Omega \times R^+} = 0 \\ u_i(x,0) = u_{i0}(x), \dfrac{\partial u_i}{\partial t}(x,0) = u_{i1}(x) \end{cases} \tag{1.1}$$

here

$$u_i(t) \in H_0^1(\Omega), \quad v_i(t) \in L^2(\Omega), \quad i = 1, 2, \cdots, n$$

$x \in \Omega, \Omega$ is a bounded open domain with sufficiently smooth boundary, $\Omega \subset R^m (m = 1, 2, 3)$, damping constant $\gamma > 0$, coupling constant $K > 0$, constant $\beta > 0$.

We write (1.1) as an evolution equation in Hilbert space $E = (H_0^1(\Omega))^n \times (L^2(\Omega))^n$:

$$\begin{cases} \dot{u} = v \\ \dot{v} = (dB + KA)u - \gamma v + G(t,u) \\ u(0) = u_0, v(0) = v_0 \end{cases} \tag{1.2}$$

here

$$u = u(t) = (u_1(t), \cdots, u_n(t))^T, \quad v = v(t) = (v_1(t), \cdots, v_n(t))^T$$
$$u_i(t) \in H_0^1(\Omega), \quad v_i(t) \in L^2(\Omega), \quad i = 1, 2, \cdots, n$$

$$B = \begin{pmatrix} \triangle & & & \\ & \triangle & & \\ & & \ddots & \\ & & & \triangle \end{pmatrix}, \quad A = \begin{pmatrix} -1 & 1 & & & \\ 1 & -2 & 1 & & \\ & & \ddots & & \\ & & 1 & -2 & 1 \\ & & & 1 & -1 \end{pmatrix}$$

$$G(t,u) = (-\beta \sin u_1 + f_1, \cdots, -\beta \sin u_n + f_n)^T$$

Let $V = (H_0^1(\Omega))^n, H = (L^2(\Omega))^n$, suppose $f = (f_1, \cdots, f_n) \in C(R^+, V)$ and f is T periodic about t. Suppose $u_0 \in V, v_0 \in H$. Let

$$Y = (u, v)^T, \quad C = \begin{pmatrix} 0 & I \\ dB + KA & -\gamma I \end{pmatrix}, \quad F(t, Y) = (0, G(t,u))^T$$