J. Partial Diff. Eqs. 14(2001), 97–104

Vol.14, No.2

TIME-PERIODIC SOLUTIONS TO THE GINZBURG-LANDAU-BBM EQUATIONS

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(Received Sept. 20, 1999; revised June 28, 2000)

Abstract In this paper, the existence and uniqueness of the time-periodic solutions to the Ginzburg-Landau-BBM equations are proved by using *a priori* estimates and Leray-Schauder fixed point theorem.
Key Words Time-periodic solution; *a priori* estimate; Leray-Schauder fixed point theorem; Ginzburg-Landau-BBM equations.
1991 MR Subject Classification 35B10, 35A05.
Chinese Library Classification O175.2.

1. Introduction

In order to understand the spatial behavior of the solutions to the Ginzburg-Landau equation coupled with BBM equation, in this paper we will consider the existence and uniqueness of the time-periodic solutions for the following systems:

$$\varepsilon_t + \mu\varepsilon - (\alpha_1 + i\alpha_2)\varepsilon_{xx} + (\beta_1 + i\beta_2)|\varepsilon|^2\varepsilon - i\delta n\varepsilon = g \tag{1.1}$$

$$n_t + f(n)_x + \gamma n - \nu n_{xx} - n_{xxt} + |\varepsilon|_x^2 = 0 \tag{1.2}$$

$$\varepsilon(x+l,t) = \varepsilon(x,t), \quad n(x+l,t) = n(x,t) \tag{1.3}$$

with the ω-periodic conditions

$$\varepsilon(x,t+\omega) = \varepsilon(x,t), \quad n(x,t+\omega) = n(x,t) \tag{1.4}$$

where $\varepsilon(x,t)$ is a complex function, $n(x,t)$ is a real scalar function, $f(n)$ is a nonlinear function, $\mu, \alpha_1, \beta_1, \delta, \gamma, \nu, l > 0$ are real constants, $g(x,t)$ is a given real function.

This problem describes the nonlinear interactions between the Langmuir wave and the ion acoustic wave in plasma physics, $\varepsilon(x,t)$ denotes electric field, $n(x,t)$ is the density [1–3]. The global existence of the smooth solutions for the problem (1.1)–(1.3) with the initial conditions has been obtained by Guo and Jiang in [4]. Here, by using *a priori* estimates and Leray-Schauder fixed point theorem, we will show the existence of approximate solutions (ε_N, n_N) of the problem (1.1)–(1.4), establish the uniform boundedness of the norm $\|\varepsilon_N(t)\|$ and $\|n_N(t)\|$, by standard compactness arguments to get the convergence of the approximate solutions, obtain the existence and uniqueness of the periodic solutions for the problem (1.1)–(1.4).

2. Existence of Approximate Solutions

Let $\phi_j(x)(j=1,2,\cdots)$ be the normalized eigenfunctions of the equation $u_{xx}+\Lambda u = 0$, with the periodic condition corresponding to eigenvalues $\Lambda_j(j=1,2,\cdots)$. $\{\phi_j\}$ forms a normalized orthogonal system of eigenfunctions.

Let $W_N = Span\{\phi_1, \phi_2, \cdots, \phi_N\}$. By [5], we know that for any

$$u_N(t) = \sum_{j=1}^{N} a_{jN}(t)\phi_j, \quad v_N(t) = \sum_{j=1}^{N} b_{jN}(t)\phi_j \in C^1(\omega, W_N) \times C^1(\omega, W_N)$$

there exists a unique ω-periodic solution

$$\varepsilon_N(x,t) = \sum_{j=1}^{N} p_{jN}(t)\phi_j(x), \quad n_N(x,t) = \sum_{j=1}^{N} q_{jN}(t)\phi_j(x) \in C^1(\omega, W_N) \times C^1(\omega, W_N)$$

for the following linear equations

$$(\varepsilon_{Nt} + \mu\varepsilon_N - (\alpha_1 + i\alpha_2)\varepsilon_{Nxx} - g, \phi_j) = (-(\beta_1 + i\beta_2)|u_N|^2 u_N + i\delta u_N v_N, \phi_j) \quad (2.1)$$

$$(n_{Nt} + \gamma n_N - \nu n_{Nxx} - n_{Nxxt}, \phi_j) = (-f(v_N)_x - |u_N|^2_x, \phi_j) \quad (2.2)$$

$$\varepsilon_N(x+l,t) = \varepsilon_N(x,t), \quad n_N(x+l,t) = n_N(x,t) \quad (2.3)$$

$$\varepsilon_N(t+\omega) = \varepsilon_N(t), \quad n_N(t+\omega) = n_N(t) \quad (2.4)$$

For the mapping $F : (u_N, v_N) \to (\varepsilon_N, n_N)$ is continuous and compact in $C^1(\omega, W_N) \times C^1(\omega, W_N)$, we may use Kato H.[5] idea to prove the existence of the solution for problem (2.1)–(2.4) through the Leray-Schauder fixed point theorem. It is sufficient to show the boundedness

$$\sup_{0\le t\le\omega} (\|\varepsilon_N(t)\|_{H^2} + \|n_N(t)\|_{H^2}) \le C$$

for the possible solution of (2.1)–(2.4) with the nonlinear terms in the right side of (2.1) and (2.2) multiplied by $\lambda(0 \le \lambda \le 1)$, where C is a constant independent of λ. That is we shall consider about the following equations:

$$(\varepsilon_{Nt} + \mu\varepsilon_N - (\alpha_1 + i\alpha_2)\varepsilon_{Nxx} + \lambda(\beta_1 + i\beta_2)|\varepsilon_N|^2\varepsilon_N - i\lambda\delta n_N\varepsilon_N - g, \phi_j) = 0 \quad (2.1a)$$

$$(n_{Nt} + \lambda f(n_N)_x + \gamma n_N - \nu n_{Nxx} - n_{Nxxt} + \lambda|\varepsilon_N|^2_x, \phi_j) = 0 \quad (2.2a)$$

$$\varepsilon_N(x+l,t) = \varepsilon_N(x,t), \quad n_N(x+l,t) = n_N(x,t) \quad (2.3a)$$

$$\varepsilon_N(t+\omega) = \varepsilon_N(t), \quad n_N(t+\omega) = n_N(t) \quad (2.4a)$$