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ON THE EXISTENCE OF POSITIVE RADIAL SOLUTIONS FOR A CLASS OF QUASILINEAR ELLIPTIC SYSTEMS*

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Abstract We study the existence of positive radial solutions for a class of quasilinear elliptic systems in a ball domains *via* the blowing up argument and degree theory. The main results of the present paper are new and extend the previously known results.
Key Words Quasilinear elliptic systems; positive radial solutions; blow up argument; degree theory.
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1. Introduction

In this paper we shall discuss the existence of positive radial solutions of the problem

$$-\operatorname{div}(|Du|^{p-2}Du) = f(x,u,v,w) \text{ in } \Omega \subset R^N \tag{1.1}$$
$$-\operatorname{div}(|Dv|^{q-2}Dv) = g(x,u,v,w) \text{ in } \Omega \subset R^N \tag{1.2}$$
$$-\operatorname{div}(|Dw|^{m-2}Dw) = h(x,u,v,w) \text{ in } \Omega \subset R^N \tag{1.3}$$
$$u = v = w = 0 \text{ on } \partial\Omega \tag{1.4}$$

where $\Omega = B_R$ is the ball centered at zero and radius R in R^N. Also let $p,q,m > 1$ and $f,g,h : \Omega \times R^N \to [0,+\infty)$ be given functions which we specify later.

In recent years, the problems of type (1.1)–(1.4) with two quasilinear elliptic equations

$$-\operatorname{div}(|Du|^{p-2}Du) = f(x,u,v) \quad \text{in} \quad \Omega \subset R^N \tag{1.5}$$
$$-\operatorname{div}(|Dv|^{q-2}Dv) = g(x,u,v) \quad \text{in} \quad \Omega \subset R^N \tag{1.6}$$
$$u = v = 0 \quad \text{on} \quad \partial\Omega \tag{1.7}$$

have been studied by many authors using different approaches. See, for example, [1–11].

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In the case when there exists a Hamiltonian function $H : \Omega \times R^2 \to R$ such that

$$f(x,t,s) = H_s(x,s,t), \quad g(x,s,t) = H_t(x,s,t)$$

the problem (1.5)–(1.7) has a variational structure and thus under some suitable hypotheses on H it can be studied by variational methods. For the results of this type the interested reader may refer to [1] and the references therein.

The problem (1.5)–(1.7) was also studied in a quite different method in [12] and [13] when $p = 2, q = 2$. In [12], *a-priori* bounds of positive solutions are proved in the case that Ω is a ball and for a large class of Hamiltonians of the form

$$H(u,v) = F(u) + G(v)$$

while in [13] it is assumed that Ω is a convex domain and $F'' \geq 0, G'' \geq 0$. In both papers the *a-priori* bounds are obtained by using the integral indentities proved in [14, 15] and [16]. For the additional results on *a-priori* bounds for positive solutions of semilinear elliptic equations and systems see [17–19] and the references therein.

In the recent papers [3,6], Clement, Manasevich, Mitidieri and Guo discussed the existence of positive radial solutions of the problem (1.5)–(1.7) when $p, q > 1, f(x,u,v) = a(|x|)|v|^{\delta-1}v, g(x,u,v) = b(|x|)|u|^{\mu-1}u$ with Ω being a ball or an annular. In the present paper we shall study the existence of positive radial solutions of the problem (1.1)–(1.4) in a ball domain Ω contained in R^N with more general functions $f(x,u,v,w) = a(|x|)u^{\alpha_1}v^{\beta_1}w^{\gamma_1}, g(x,u,v,w) = b(|x|)u^{\alpha_2}v^{\beta_2}w^{\gamma_2}, h(x,u,v,w) = c(|x|)u^{\alpha_3}v^{\beta_3}w^{\gamma_3}$, complementing the results of [3,6]. To prove the existence of positive radial solutions of (1.1)–(1.4) we first need to study some nonexistence results for a class of quasilinear elliptic problem in R^N, then establish *a-priori* bounds for positive radial solutions for this system by the blowing up argument as in [20], finally using degree theory. The main result is

Theorem A Consider the system

$$-\operatorname{div}(|Du|^{p-2}Du) = a(|x|)u^{\alpha_1}v^{\beta_1}w^{\gamma_1} \text{ in } B_R \subset R^N \tag{1.8}$$

$$-\operatorname{div}(|Dv|^{q-2}Dv) = b(|x|)u^{\alpha_2}v^{\beta_2}w^{\gamma_2} \text{ in } B_R \subset R^N \tag{1.9}$$

$$-\operatorname{div}(|Dw|^{m-2}Dw) = c(|x|)u^{\alpha_3}v^{\beta_3}w^{\gamma_3} \text{ in } B_R \subset R^N \tag{1.10}$$

$$u = v = w = 0 \text{ on } \partial B_R \tag{1.11}$$

where $r = |x|, a, b, c : [0, +\infty) \to (0, +\infty)$ are continuous functions such that

$$\bar{a} = \min_{s\in[0,+\infty)} a(s) > 0, \quad \bar{b} = \min_{s\in[0,+\infty)} b(s) > 0, \quad \bar{c} = \min_{s\in[0,+\infty)} c(s) > 0$$

Suppose that

(i) $\alpha_1 > p-1, \beta_2 > q-1, \gamma_3 > m-1$ with $p, q, m > 1, \alpha_2, \alpha_3, \beta_1, \beta_3, \gamma_1, \gamma_2 \geq 0$;