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GLOBAL $W^{2,p}$ SOLUTIONS OF GBBM EQUATIONS ON UNBOUNDED DOMAIN*

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Abstract In this paper we study the initial boundary value problem of GBBM equations on unbounded domain

$$u_t - \Delta u_t = \operatorname{div} f(u)$$
$$u(x,0) = u_0(x)$$
$$u|_{\partial\Omega} = 0$$

and corresponding Cauchy problem. Under the conditions: $f(s) \in C^1$ and satisfies

(H) $$|f'(s)| \leq C|s|^\gamma, \quad 0 \leq \gamma \leq \frac{2}{n-2} \text{ if } n \geq 3; \quad 0 \leq \gamma < \infty \text{ if } n = 2$$

$u_0(x) \in W^{2,p}(\Omega) \cap W^{2,2}(\Omega) \cap W_0^{1,p}(\Omega)(W^{2,p}(R^n) \cap W^{2,2}(R^n)$ for Cauchy problem), $2 \leq p < \infty$, we obtain the existence and uniqueness of global solution $u(x,t) \in W^{1,\infty}(0,T; W^{2,p}(\Omega) \cap W^{2,2}(\Omega) \cap W_0^{1,p}(\Omega))(W^{1,\infty}(0,T; W^{2,p}(R^n) \cap W^{2,2}(R^n))$ for Cauchy problem), so the results of [1] and [2] are generalized and improved in essential.

Key Words GBBM equation; unbounded domain; global $W^{2,p}$ solutions; existence.

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1. Introduction

In [1] Goldstein et al studied the initial boundary value problem of GBBM equations on unbounded domain

$$u_t - \Delta u_t = \operatorname{div} f(u) \tag{1.1}$$
$$u(x,0) = u_0(x) \tag{1.2}$$
$$u|_{\partial\Omega} = 0 \tag{1.3}$$

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and corresponding Cauchy problem. Under the conditions: $f(s) \in C^2, f'(0) = 0$ and satisfies

$$\text{(H')} \qquad |f'(s)| \le C(1 + |s|^\gamma), \quad 0 < \gamma \le \frac{2}{n-2} \text{ if } n \ge 3; \quad 0 < \gamma < \infty \text{ if } n = 2,$$

$u_0(x) \in W^{2,p}(\Omega) \cap W^{2,2}(\Omega) \cap W_0^{1,p}(\Omega)$, where

$$\max\left\{\frac{n}{2}, 1\right\} < p < n \tag{1.4}$$

they proved the existence of global $W^{2,p}$ solution.

In [2] Guo Boling and Miao Changxing again studied the above problem, under the same conditions on $f(s)$ and $u_0(x)$, but the condition (1.4) was replaced by

$$\max\left\{\frac{n}{2}, 1\right\} < p < \infty$$

they obtained the same conclusion as [1].

In [3] Liu Yacheng and Wan Weiming studied the problem (1.1)–(1.3) on bounded domain, under the conditions: $f(s) \in C^1$ and satisfies (H'), $u_0(x) \in W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega), 2 \le p < \infty$, but without assumption $n < 2p$, obtained the existence and uniqueness of global solution $u(x,t) \in W^{1,\infty}(0,T; W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega)), \forall T > 0$, so the results of [1],[2] and [4] on bounded domain was improved and generalized.

As indicated in [3], the condition $n < 2p$ is very harsh, for example, for the most important case $p = 2$, the space dimensions n must satisfy $n \le 3$.

In this paper we still study the problem (1.1)–(1.3) on unbounded domain and corresponding Cauchy problem based on [3], under the conditions: $f(s) \in C^1$ and satisfies

$$\text{(H)} \qquad |f'(s)| \le C|s|^\gamma, \quad 0 < \gamma \le \frac{2}{n-2} \text{ if } n \ge 3; 0 < \gamma < \infty \text{ if } n = 2$$

$u_0(x) \in W^{2,p}(\Omega) \cap W^{2,2}(\Omega) \cap W_0^{1,p}(\Omega) (W^{2,p}(R^n) \cap W^{2,2}(R^n)$ for Cauchy problem), $2 \le p < \infty$ but without assumption $n < 2p$, we obtain the existence and uniqueness of global solution $u(x,t) \in W^{1,\infty}(0,T; W^{2,p}(\Omega) \cap W^{2,2}(\Omega) \cap W_0^{1,p}(\Omega)) (W^{1,\infty}(0,T; W^{2,p}(R^n) \cap W^{2,2}(R^n))$ for Cauchy problem), $\forall T > 0$, so the results of [1] and [2] on unbounded domain are generalized and improved.

Lemma 1.1[3] *Suppose that Ω is a sufficiently smooth bounded domain, $f(s) \in C^1$ and satisfies (H'), $u_0(x) \in W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega), 2 \le p < \infty$. Then the problem (1.1)–(1.3) has a unique global solution $u(x,t) \in W^{1,\infty}(0,T; W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega)), \forall T > 0$.*

2. Global $W^{2,2}$ Solutions

First, we consider the Cauchy problem

$$u_t - \Delta u_t = \operatorname{div} f(u), \quad x \in R^n, t > 0 \tag{2.1}$$

$$u(x,0) = u_0(x), \quad x \in R^n \tag{2.2}$$