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THE GLOBAL ATTRACTORS FOR A DAMPED GENERALIZED COUPLED NONLINEAR WAVE EQUATIONS

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Abstract The existence of global attractors for the periodic initial value problem of damped generalized coupled nonlinear wave equations is proved. We also get the estimates of the upper bounds of Hausdorff and fractal dimensions for the global attractors by means of a uniformly priori estimates for time.

Key Words Global Attractor; Hausdorff Dimension; Fractal Dimension.
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1. Introduction

In [1], the authors established the unique existence of the smooth solution for the following coupled nonlinear equations

$$u_t = u_{xxx} + buu_x + 2vv_x, \tag{1.1}$$

$$v_t = 2(uv)_x. \tag{1.2}$$

These were proposed to describe the interaction process of internal long waves. In [2], Ito M. proposed a recursion operator by which he inferred that the equations (1.1) and (1.2) possess infinitely many symmetries and constants of motion. In [3], P.F.He established the existence of a smooth solution to the system of coupled nonlinear KdV equation [4]

$$u_t = a(u_{xxx} + buu_x) + 2bvv_x, \tag{1.3}$$

$$v_t = -v_{xxx} - 3uv_x, \tag{1.4}$$

where a and b are constants.

We remark that M. E. Schonbek [5] dealt with a very similar system of coupled nonlinear equation [6]

$$u_t = u_{xxx} - uu_x - v_x, \tag{1.5}$$

$$v_t = -(uv)_x. \tag{1.6}$$

The global existence of a weak solution was established via the technique of parabolic regularization and Dunford's theorem on weakly sequentially compact L^1 sets.

In this paper, we consider the following periodic initial value problem of damped coupled nonlinear wave equations

$$u_t + f(u)_x - \alpha u_{xx} + \beta u_{xxx} + 2vv_x = G_1(u, v) + h_1(x), \tag{1.7}$$

$$v_t - \gamma v_{xx} + (2uv)_x + g(v)_x = G_2(u, v) + h_2(x), \tag{1.8}$$

$$u(x + D, t) = u(x - D, t), v(x + D, t) = v(x - D, t), x \in R, t \geq 0, \tag{1.9}$$

$$u(x, 0) = u_0(x), v(x, 0) = v_0(x), x \in R, \tag{1.10}$$

where $D > 0$, $\alpha > 0$, $\beta \neq 0$, $\gamma > 0$ are real numbers, and $\int_{-D}^{D} u(x,t)dx = 0, \int_{-D}^{D} v(x,t)dx = 0$. We establish the t-independent a priori estimates of the problem (1.7)-(1.10) and get the estimate of upper bounds of Hausdorff and fractal dimensions for the global attractor.

To simplify the notation in this paper, we shall denote by $\|\cdot\|$ the norm $\|\cdot\|_{L_2}$, by $\|\cdot\|_p$ the norm $\|\cdot\|_{L^p}$, by $\|\cdot\|_\infty$ the norm $\|\cdot\|_{L^\infty}$, by $\|\cdot\|_m$ the norm $\|\cdot\|_{H^m}$, $\Omega = (-D, D)$.

2. t-independent A Priori Estimates of Problem (1.7)-(1.10)

Lemma 1 *Suppose that*

(1) $G_i(0,0) = 0 \quad (i = 1, 2)$, $\quad (\xi, \eta)\begin{pmatrix} -G_{1u} & -G_{2u} \\ -G_{1v} & -G_{2v} \end{pmatrix}\begin{pmatrix} \xi \\ \eta \end{pmatrix} \geq b_0(|\xi|^2 + |\eta|^2)$, $(\xi, \eta) \in \mathrm{R}^2$, $b_0 > 0$ *is a constant,*

(2) $u_0 \in L^2(\Omega), v_0 \in L^2(\Omega), h_i(x) \in L^2(\Omega)(i = 1, 2), \Omega = (-D, D)$.

Then for the smooth solution of the problem (1.7)-(1.10), we have the following estimate

$$\|u(\cdot, t)\|^2 + \|v(\cdot, t)\|^2 \leq e^{-b_0 t}(\|u_0\|^2 + \|v_0\|^2) + \frac{1}{b_0^2}(1 - e^{-b_0 t})(\|h_1\|^2 + \|h_2\|^2). \tag{2.1}$$

Furthermore, we have

$$\overline{\lim_{t\to\infty}}(\|u(\cdot, t)\|^2 + \|v(\cdot, t)\|^2) \leq \frac{1}{b_0^2}(\|h_1\|^2 + \|h_2\|^2) = E_0, \tag{2.2}$$

$$\overline{\lim_{t\to\infty}}\frac{1}{t}\int_0^t [\alpha\|u_x(\cdot, \tau)\|^2 + \gamma\|v_x(\cdot, \tau)\|^2]d\tau \leq \frac{1}{b_0^2}(\|h_1\|^2 + \|h_2\|^2). \tag{2.3}$$

Proof Taking the inner product of (1.7) with u, (1.8) with v, then we have

$$(u, u_t + f(u)_x - \alpha u_{xx} + \beta u_{xxx} + 2vv_x) = (u, G_1(u, v) + h_1(x)), \tag{2.4}$$

$$(v, v_t - \gamma v_{xx} + (2uv)_x + g(v)_x) = (v, G_2(u, v) + h_2(x)), \tag{2.5}$$