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LONG TIME BEHAVIOR OF THE DISSIPATIVE GENERALIZED SYMMETRIC REGULARIZED LONG WAVE EQUATIONS

Shang Yadong, Guo Boling and Fang Shaomei
(Laboratory of Computational Physics, Institute of Applied Physics and Computational Mathematics, P.O.Box 8009, Beijing 100088, China)
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Abstract This paper deals with the long time behavior of solutions for the dissipative generalized symmetric regularized long wave equations. We show the existence of global weak attractors for the periodic initial value problem of the equations in $H^1 \times L^2$. The finite dimensionality of the global attractors is also established.
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1. Introduction

A symmetric version of regularized long wave equation (SRLWE)

$$u_{xxt} - u_t = \rho_x + uu_x, \tag{1.1}$$

$$\rho_t + u_x = 0, \tag{1.2}$$

has been proposed as a model for propagation of weakly nonlinear ion acoustic and space-charge waves[1]. The $sech^2$ solitary wave solutions, the four invariants and some numerical results have been obtained in [1]. Obviously, eliminating ρ from (1.1), we get a class of symmetric regularized long wave equation (SRLWE)

$$u_{tt} - u_{xx} + \frac{1}{2}(u^2)_{xt} - u_{xxtt} = 0. \tag{1.3}$$

The SRLW equation (1.3) is explicitly symmetry in the x and t derivatives and is very similar to the regularized long wave equation which describes shallow water waves and plasma drift waves[2-3]. The SRLW equation (1.1)—(1.2) or (1.3) arises also in many other areas of mathematical physics. Numerical investigation indicated that interactions of solitary waves are inelastic [4], thus the solitary wave of the SRLW equation is not soliton. More recently, Chen Lin ([5]) studied the orbital stability and instability of solitary wave solutions of the generalized SRLW equations. The research on the well-posedness and numerical methods for the equation has aroused more and more interest. In [6] Guo Boling studied the existence, uniqueness and regularity of the periodic initial value problem for a class of the generalized SRLW equations and obtained the error estimates of the spectral approximation. Miao Chenxia [7] considered the initial boundary value problem for symmetric regularized long wave equations with non homogenous boundary value.

In real processes, viscosity, as well as dispersion, plays an important role. Therefore, it is more significant to study the behavior (especially the large time behavior)of the dissipative symmetric regularized long wave equations with damping term

$$u_{xxt} - u_t + \nu u_{xx} = \rho_x + uu_x, \tag{1.4}$$

$$\rho_t + u_x + \gamma\rho = 0. \tag{1.5}$$

where γ, ν are positive constants, which is a reasonable model to render essential phenomena of nonlinear ion acoustic wave motion when take account of dissipation.

In this paper, we consider the following periodic initial value problem for the dissipative generalized symmetric regularized long wave equations with damping term

$$u_t - \nu u_{xx} + \rho_x + f(u)_x - u_{xxt} = g_1(x), (x,t) \in \mathrm{R} \times \mathrm{R}^+, \tag{1.6}$$

$$\rho_t + u_x + \gamma\rho = g_2(x), (x,t) \in \mathrm{R} \times \mathrm{R}^+, \tag{1.7}$$

$$u(x+D,t) = u(x-D,t), \rho(x+D,t) = \rho(x-D,t), x \in \mathrm{R}, t \geq 0, \tag{1.8}$$

$$u(x,0) = u_0(x), \rho(x,0) = \rho_0(x), \quad x \in \mathrm{R}, \tag{1.9}$$

where $D > 0, \gamma, \nu > 0$ are positive constants, $f : \mathrm{R} \to \mathrm{R}$ are C^∞functions, $g_1(x), g_2(x) \in L^2_{per}(\Omega), \Omega = (-D, D)$, we establish the t-independent a priori estimates of the problem (1.6)–(1.9), then we prove the existence of global attractor of the problem (1.6)–(1.9) in $H^1(\Omega) \times L^2(\Omega)$, and establish the finite-dimensionality of Hausdorff and fractal dimension for the global attractor. Since the dynamical system $S(t)$ defined by (1.6) and (1.7) is not compact in $H^1(\Omega) \times L^2(\Omega)$, we cannot construct the global attractor by the method introduced by Temam [8] or Constantin, Foias and Temam[9]. We here employ the techniques developed by Ghidaglia[10] to show the existence of finite dimensional global weak attractor for (1.6)–(1.7) in $H^1(\Omega) \times L^2(\Omega)$. For this purpose, it is necessary that the semigroup $S(t)$ should be weakly continuous in $H^1(\Omega) \times L^2(\Omega)$ for every $t > 0$. We will establish the weak continuity of $S(t)$ in $H^1(\Omega) \times L^2(\Omega)$ by applying a direct method.

The outline of this article is as follows. In Section 2, we show that the solution semigroup $S(t)$ is weakly continuous in $H^1(\Omega) \times L^2(\Omega)$ for every $t > 0$. In Section 3, we derive the uniform a priori estimates in time on the solution of the equations (1.6)–(1.7) in $H^1(\Omega) \times L^2(\Omega)$. Then we show that the existence of global weak attractor for the equations (1.6)–(1.7) in $H^1(\Omega) \times L^2(\Omega)$. The finite dimensionality of the global weak attractor is also deduced.