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APPROXIMATION TO NONLINEAR SCHRÖDINGER EQUATION OF THE COMPLEX GENERALIZED GINZBURG-LANDAU EQUATION

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Abstract In this paper, we prove that in the inviscid limit the solution of the generalized derivative Ginzburg-Landau equations converges to the solution of derivative nonlinear Schrödinger equation, we also give the convergence rates for the difference of the solution.
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1. Introduction

Derivative Ginzburg-Landau equation appeared in many physical problems, it was derived from instability waves in hydrodynamics such as the nonlinear growth of Rayleigh-Benard convective rolls, the appearance of Taylor Vortices in the couette flow among counter-rotating cylinders, the development of Tallmien-schlichting waves in plane poiseuille flows, the transition to turbulence in chemical reactions ([1–4]).

The generalized derivative Ginzburg-Landau equation for the function $u(x,t)$ is given by

$$u_t = (a + i\nu)u_{xx} + \sigma u + \beta u^2\bar{u}_x + \gamma|u|^2 u_x - (b + i\mu)f(u), \tag{1.1}$$

where a, b, ν, μ, σ, β, γ are real constants. We consider this equation either in the whole space $\Omega = R$ under the initial condition

$$u(x,0) = u_0(x), \qquad x \in \Omega, \tag{1.2}$$

or in $\Omega = [0,l]$, under the initial condition (1.2) and periodic boundary condition

$$u(x + l, t) = u(x,t), \qquad x \in R, \quad t \geq 0. \tag{1.3}$$

The equation (1.1) reduces to the derivative nonlinear Schrödinger equation for $a = b = 0$

$$u_t = i\nu u_{xx} + \sigma u + \beta u^2 \bar{u}_x + \gamma |u|^2 u_x - i\mu f(u). \tag{1.4}$$

In [5] Duan and Holmes and Titi have proved the existence and uniqueness of global solution on bounded domain for the equation (1.1), in [6] Duan and Holmes have proved the posedness of Cauchy problem for the equation (1.1), in [7] Guo and Gao have proved the existence of global attractor of periodic initial boundary problem for the equation (1.1). Tan and Guo in [8] proved the existence and uniqueness of the smooth solution to the Cauchy problem and periodic boundary value problem for the equation (1.4). When $\beta = \gamma = 0$, the reference [9] proved the inviscid limit $a \to 0$, $b \to 0$ in the equation (1.1) and give optimal convergence rates for the difference of solutions to the equation (1.1) and (1.4). In this paper, we will prove that in the inviscid limit the solution of equation (1.1) converges to the solution of the equation (1.4) and give optimal convergence rate for the difference the solution to the equation (1.1) and (1.4). We denote $\|\cdot\|_{L^p(\Omega)}$ by $\|\cdot\|_p$.

2. Preliminaries and Main Results

We suppose that

$$0 < a < a_0, \qquad 0 < b < b_0; \tag{2.1}$$

$$\nu > 0, \quad \sigma \geq 0, \quad \mu \in R; \tag{2.2}$$

there exist constants Γ_1 and Γ_2 such that

$$\Gamma_1 s^2 \leq g(s) \leq \Gamma_2(1 + s^2), \qquad g'(s) \geq 0, \tag{2.3}$$

where $f(u) = g(|u|^2)u$,

$$16\mu\nu\Gamma_1 + (\beta + \gamma)(5\beta - 3\gamma) > 0. \tag{2.4}$$

Let $\varepsilon = (a, b)$, $a^{-1}b$ be bounded as $\varepsilon \to 0$. Let $V = H^1(\Omega)$, $T > 0$, and let u_ε be the solution to the problem (1.1), (1.2) or (1.1), (1.2), (1.3) in $\Omega \times (0, T)$.

The main results are as follows:

Theorem 2.1 *Let the above assumptions hold, $u_0 \in H^1(\Omega)$, $\|u_0\|_2$ is small enough, then there exists a subsequence of (u_ε) such that, as $\varepsilon \to 0$, u_ε converges to a function u which solves the problem* (1.4), (1.2) *or* (1.4), (1.2), (1.3). *The convergence is strong in the space $L^\infty(0,T;L^r_{loc}(\Omega))$, where $2 \leq r < \infty$, weak* in the spaces $L^\infty(0,T;H^1(\Omega))$ and $W^{1,\infty}(0, T; V^*)$.*

Theorem 2.2 *Suppose that the conditions of* (2.1)—(2.4) *hold, let $u_0 \in H^2(\Omega)$, and $\|u_0\|_2$ is small enough, then there exists a subsequence of (u_ε), such that as $\varepsilon \to 0$, u_ε converges to a function u which solves the problem* (1.4), (1.2) *or* (1.4), (1.2), (1.3) *and*

$$u_\varepsilon \to u \quad \text{in } L^\infty(0, T; H^2(\Omega)) \quad \text{weakly-}^*;$$