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INTEGRAL AVERAGING TECHNIQUE FOR OSCILLATION OF ELLIPTIC EQUATIONS OF SECOND ORDER

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Abstract The elliptic differential equations of second order

$$\sum_{i,j=1}^{n} D_i[A_{ij}(x,y)D_jy] + P(x,y) + Q(x,y,\nabla y) = e(x), \quad x \in \Omega.$$

will be considered in an exterior domain $\Omega \subset R^n$, $n \geq 2$. Some oscillation criteria are given by integral averaging technique.
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1. Introduction

The oscillation of the solutions of second order elliptic differential equations has been intensively studied in recent years by many authors [see, for example, [1-10] and the references cited therein]. However, as far as we know, there are few results concerning nonlinear elliptic differential equations of second order by using integral averaging technique. Motivated by this fact, we intend here to study the oscillatory behavior of solutions of nonlinear elliptic differential equations of second order

$$\sum_{i,j=1}^{n} D_i\left[A_{ij}(x,y)D_jy\right] + P(x,y) + Q(x,y,\nabla y) = e(x), \quad x \in \Omega, \qquad (E)$$

where Ω is an exterior domain in R^n and functions $A_{i,j}$, P, Q, e are to be specified in the following text. Using integral averaging and completing square technique (see, [11-13]

) which has here been developed further, we give sufficient conditions for any proper solution $y(x)$ of Eq.(E) either to satisfy $\liminf_{|x|\to\infty}|y(x)|=0$ or to be oscillatory. The obtained theorems here extend and improve the main results [4] and [7-10]. Moreover, some examples are given to illustrate the advantages of the obtained results.

As usual, $R^+=(0,\infty)$, $R^-=(-\infty,0)$. $x=(x_1,x_2,\cdots,x_n)\in R^n$, $|x|=[\sum_{i=1}^n {x_i}^2]^{\frac{1}{2}}$, and differentiation with respect to x_i are denoted by D_i, $(i=1,2\cdots,n)$. $\nabla=(D_1,D_2,\cdots,D_n)$. $S_a=\{x\in R^n:|x|=a\}$, $G_a=\{x\in R^n:|x|>a\}$, $(a>0)$. The measure on S_a and S_1 will be denoted by S and ω, respectively. Thus $dS=a^{n-1}d\omega$. The outward unit normal ν to S_a at $x\in S_a$ has components $\nu_i(x)=x_i/|x|$, $(i=1,2,\cdots,n)$.

Throughout this paper, Eq.(E) is to be considered in an exterior domain $\Omega\subset R^n$ (ie. $G_{t_0}\subset\Omega$ for some positive number t_0) subject to the following assumptions.

(A_1) $A=(A_{ij})_{n\times n}$ is a real symmetric positive definite function matrix (ellipticity condition) with $A_{i,j}\in C_{loc}^{1+\mu}(\Omega\times R,R^+)$, $\mu\in(0,1)$, $(i,j=1,2,\cdots,n)$, and let $\lambda_{\max}(x,y)$ denote the largest (necessary positive) eigenvalue of the matrix A. Assume that there exists a function $\lambda\in C[R^+\times R,R^+]$ such that

$$\lambda(r,y)=\max_{|x|=r}\lambda_{\max}(x,y),\quad (r>0);$$

(A_2) $P\in C_{loc}^{\mu}(\Omega\times R,R)$, $Q\in C_{loc}^{\mu}(\Omega\times R\times R^n,R)$, $\mu\in(0,1)$ such that for $y\neq 0$

$$yP(x,y)\geq yp(x)f_1(y),\quad yQ(x,y,\nabla y)\geq q(x)yf_2(y)g(\nabla y),$$

where $p\in C(\Omega,R)$, $q\in C(\Omega,R-R^-)$, $f_1\in C'(R,R)$, $f_2\in C(R,R)$ and $g\in C(R^n,R)$ such that

(i) $xf_1(x)>0$, $xf_2(x)\geq 0$ and $f_2(x)/f_1(x)\geq k\geq 0$ for $x\neq 0$;

(ii) $g(\nabla y)\geq C$ for some $C>0$;

(A_3) $e\in C_{loc}^{\mu}(\Omega)$, $\mu\in(0,1)$.

Definition 1 *For $\Omega\subset R^n$ and $\mu\in(0,1)$, a function $y(x)\in C_{loc}^{2+\mu}(\Omega)$ which satisfies Eq.(E) for all $x\in\Omega$ is called a solution of Eq.(E) in Ω.*

We often assume that the solution of Eq.(E) exists in an exterior domain Ω under the above assumption (see [14]).

Definition 2 *A proper solution $y(x)$ of Eq.(E) is called oscillatory in Ω whenever the set $\{x\in\Omega:y(x)=0\}$ is unbounded. Eq.(E) is called oscillatory in Ω whenever every proper solution of Eq.(E) is oscillatory in Ω.*

Following Philos [13], let us introduce now the class of functions $\Re$ which will be extensively used in the sequel.

Definition 3 *Let $D_0=\{(t,s):t>s\geq t_0\}$ and $D=\{(t,s):t\geq s\geq t_0\}$. We say that a function $H\in C(D,R)$ belongs to a function class $\Re$ (or $H\in\Re$, for short) if*

(i) *$H(t,t)=0$ for $t\geq t_0$; $H(t,s)>0$ for $(t,s)\in D_0$;*

(ii) *H has a continuous and nonpositive partial derivative on D_0 with respect to the second variables, and there exists a function $h\in C[D,R]$ such that*

$$-\frac{\partial H}{\partial s}(t,s)=h(t,s)\sqrt{H(t,s)}\quad \text{for all}\quad (t,s)\in D_0.$$