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UPWIND DISCONTINUOUS GALERKIN METHODS FOR TWO DIMENSIONAL NEUTRON TRANSPORT EQUATIONS*

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Abstract In this paper the upwind discontinuous Galerkin methods with triangle meshes for two dimensional neutron transport equations will be studied. The stability for both of the semi-discrete and full-discrete method will be proved.

Key Words Upwind discontinuous Galerkin methods; Neutron transport equations; Stability.

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1. Introduction

Many papers have been devoted to the discontinuous finite element methods for neutron transport equations. In [1] the neutron transport equations with coordinates in cylinderical symmetry geometries were considered, and in [2] the Galerkin methods for both space and angular variables for nonlinear neutron transport equations were studied and the uniform priori estimate for Galerkin approximate solutions was proved. In [3] the discontinuous finite element methods were first introduced for steady neutron transport equations. In [4] the generalized difference method of "upwind" type for hyperbolic equations was discussed, and the stability and convergence of the method were proved. Upwind scheme is an important numerical scheme for solving hyperbolic equations of first-order and has been studied extensively (e.g., see [5]).

In this paper the discontinuous Galerkin methods of "upwind type" for two dimensional neutron transport equations will be introduced, where the space variables are discretized by Galerkin methods with triangle meshes, and angular variables by discrete

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ordinates and source iterate methods, and time variable by implicit or Crank-Nicolson scheme. The stability of the methods will be proved.

Consider the following two dimensional neutron transport equations

$$\frac{\partial u}{\partial t}+\mu\nabla\cdot u+\Sigma_t(x)u(x,\mu,t) = \frac{\Sigma_s(x)}{2\pi}\int_D \frac{u(x,\mu',t)}{(1-|\mu'|^2)^{\frac{1}{2}}}d\mu'+f(x,t) \tag{1}$$
$$(x,\mu)\in\Omega\times D,\quad t\in(0,T)$$

with the boundary and initial conditions

$$u(x,\mu,t)=g(x,\mu,t),\quad on\ \Gamma_\mu^-$$
$$u(x,\mu,0)=u_0(x,\mu),\quad in\ \Omega\times D$$

where $\Sigma_t(x)$ and $\Sigma_s(x)$ are total cross section and scattering cross section respectively, $u(x,\mu,t)$ is an angular flux. $\Omega\subset\mathbf{R}^2$ is a convex polygon, $\Gamma=\partial\Omega$, $D=\{\mu\in\mathbf{R}^2 : |\mu|\le 1\}, \mu=(\mu_1,\mu_2)$, $\Gamma_\mu^-=\{x\in\Gamma:\ \mu\cdot\vec{n}(x)<0\}$, where $\vec{n}(x)$ is an outer unit normal vector.

Assume the following conditions hold:

$$g(x,\cdot,t)\in L^2(\Gamma_\mu^-\times[0,T]);\quad u_0(x,\cdot,t)\in L^2(\Omega\times[0,T]). \tag{A1}$$

$$\Sigma_t(x),\Sigma_s(x)\in L^\infty(\Omega), f(x,t)\in L^2(\Omega\times[0,T]). \tag{A2}$$

2. Discontuous Galerkin Methods and Main Results

First the angular variable in (1) is discretized. Let's introduce the discrete set $\Delta=\{\mu^1,\mu^2,\cdots\mu^N\}$, and approximate the integral term at the right hand of (1) with quadrature formular

$$\int_D u(\mu)(1-|\mu|^2)^{-\frac{1}{2}}d\mu\sim\sum_{\mu\in\Delta}u(\mu)\omega_\mu$$

where ω_μ is a positive weight function.

Then the discretization of the spatial variable is introduced. Let $E_h=\{K\}$ be a quasi-uniform triangulation of Ω, $diamK\le h$. Denote $V_h=\{v\in L_2(\Omega):$ $v|_K$ is linear$\}$.

Now we construct the discontuous Galerkin methods of "upwind" type.

Semi-discrete scheme: for $\mu\in\Delta$, find$u_N^h=u_N^h(\cdot,\mu,t)\in V_h$ such that

$$\frac{d}{dt}(u_N^h,v)+\sum_{K\in E_h}\left[(\mu\cdot\nabla u_N^h+\Sigma_t u_N^h,v)_K+\int_{\partial K_-}[u_N^h]v_+|\mu\cdot\vec{n}|d\sigma\right]$$
$$=\frac{1}{2\pi}(\Sigma_s U_N,v)_K+(f,v),\quad\forall v\in V_h \tag{2}$$