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EXISTENCE OF PERIODIC SOLUTIONS FOR 3-D COMPLEX GINZBERG-LANDAU EQUATION

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Abstract In this paper, the authors consider complex Ginzburg-Landau equation (CGL) in three spatial dimensions

$$u_t = \rho u + (1 + i\gamma)\Delta u - (1 + i\mu)\,|u|^{2\sigma}\,u\ + f,$$

where u is an unknown complex-value function defined in 3+1 dimensional space-time R^{3+1}, Δ is a Laplacian in R^3, $\rho > 0$, γ, μ are real parameters, $\Omega \in R^3$ is a bounded domain. By using the method of Galërkin and Faedo-Schauder fix point theorem we prove the existence of approximate solution u_N of the problem. By establishing the uniform boundedness of the norm $\|u_N\|$ and the standard compactness arguments, the convergence of the approximate solutions is considered. Moreover, the existence of the periodic solution is obtained .

Key Words complex Ginzburg-Landau equation; Galërkin method; approximate solution; time periodic solution.
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1. Introduction

The generalized complex Ginzburg-Landau (CGL) equation describes the evolution of a complex-valued $u = u\,(x, t)$ by

$$u_t = \rho u + (1 + i\gamma)\Delta u - (1 + i\mu)\,|u|^{2\sigma}\,u\ .$$

It has a long history in physics as a generic amplitude equation near the onset of instabilities that lead to chaotic dynamics in fluid mechanical systems, as well as in the theory of phase transitions and superconductivity. It is a particularity interesting model because it is a dissipative version of the nonlinear Schrödinger equation−A Hamiltonian equation which can possess solutions that form localized singularities in finite time.

Ghidaglia and Héorn [1], Doering et al [2], Promislow [3], etc. studied the finite dimensional Global attractor and related dynamic issues for the one or two spatial dimensional GLE with cubic nonlinearity ($\sigma = 1$) :

$$u_t - (1 + i\gamma)\Delta u + (1 + i\mu)\,|u|^2\,u - \rho u = 0.$$

where $i = \sqrt{-1}$, $a > 0$. and γ, μ are given real numbers. Bartuccelli, Constantin, Doering, Gibbon and Gisselfalt [4] deal with the "soft" and "hard" turbulent behavior for this equation. In [5], Bu considered the global existence of the Cauchy problem of the following 2D GLE:

$$u_t - (\nu + i\alpha)\Delta u + (\mu + i\beta)\,|u|^{2q}\,u - \gamma u = 0$$

with $q = 1$ and $q = 2$, $\alpha\beta > 0$, or $|\beta| \leq \frac{\sqrt{5}}{2}$. Doering, Gibbon and Levermore [6] investigated weak and strong solutions for this equation. Mielke [7] discussed the solution of this equation in weighted L^p space and derived some new bounds and investigated some properties of attractors. We consider the equation with non-homogeneous term in three spatial dimensions as follows

$$u_t = \rho u + (1 + i\gamma)\Delta u - (1 + i\mu)\,|u|^{2\sigma}\,u + f(x,t), \tag{1.1}$$

$$u(0,x) = u_0(x), \qquad x \in \Omega \tag{1.2}$$

with periodic boundary condition

$$\Omega = (0,L) \times (0,L) \times (0,L), \quad u \text{ is } \Omega - \text{periodic}, \tag{1.3}$$

where u is an unknown complex-value function defined in 3+1dimensional space-time R^{3+1}, Δ is a Laplacian in $R^3, \rho > 0, \gamma, \mu$ are real parameters, the function $f(x,t)$ is $\omega-$periodic in time t.

Here, by using the Galerkin method and Leray-Schauder fixed point theorem, we will show the existence of approximate solution $u_N(t)$ of the problem $(1.1) - (1.3)$. We establish the uniform boundedness of the norm $\|u_N(t)\|$, by standard compactness arguments get convergence of the approximate solution, and obtain the existence of the time periodic solution for the problem $(1.1) - (1.3)$.

Our assumptions on σ, γ, μ are (**A**):

(i) By choosing suitable γ,

$$\sigma \leq \min\left\{ \frac{\sqrt{1+\gamma^2}}{\sqrt{1+\gamma^2} - 1} - 1, \frac{1}{4}\frac{\sqrt{1+\gamma^2}}{\sqrt{1+\gamma^2} - 1} \right\};$$