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EXISTENCE OF SOLUTIONS FOR A CLASS OF QUASILINEAR ELLIPTIC SYSTEMS WITH INDEFINITE WEIGHTS*

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Abstract By use of the fibering method introduced by Pohozaev S I, the existence of positive solutions for homogeneous Dirichlet problem of a class of quasilinear elliptic systems with indefinite weights is obtained.
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1. Introduction

In this paper, we shall consider the following quasilinear elliptic problem with indefinite weights

$$\begin{cases} -\Delta_p u \equiv -\mathrm{div}(|\nabla u|^{p-2}\nabla u) = \lambda_1 m(x)|u|^{p-2}u + \dfrac{2\alpha}{\alpha+\beta}c(x)u|u|^{\alpha-2}|v|^{\beta}, & \text{in}\,\Omega, \\ -\Delta_q v = -\mathrm{div}(|\nabla v|^{q-2}\nabla v) = \mu_1 n(x)|v|^{q-2}v + \dfrac{2\beta}{\alpha+\beta}c(x)|u|^{\alpha}v|v|^{\beta-2}, & \text{in}\,\Omega, \\ u=0, \qquad v=0, & \text{on}\,\partial\Omega, \end{cases} \tag{P}$$

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where $\Omega \subset R^n$ is a bounded smooth domain, $\alpha, \beta, p > 1, q > 1$ are real constants, $m(x), n(x)$ are given functions which may change sign. We assume that

$$\begin{aligned} & m^+(x), n^+(x) \neq 0 \quad \text{and} \\ & \qquad m(x) \in L^r(\Omega) \quad \text{for some } r > \frac{n}{p} \quad \text{if } 1 < p < n \quad \text{and } r = 1 \text{ if } p > n; \\ & \qquad n(x) \in L^s(\Omega) \quad \text{for some } s > \frac{n}{q} \quad \text{if } 1 < q < n \quad \text{and } s = 1 \text{ if } q > n, \end{aligned} \tag{1.1}$$

λ_1, μ_1 are defined as

$$\begin{aligned} \lambda_1 &= \inf\left\{\int_\Omega |\nabla u|^p dx : u \in W_0^{1,p}(\Omega) \quad \text{and} \int_\Omega m(x)|u|^p dx = 1\right\}; \\ \mu_1 &= \inf\left\{\int_\Omega |\nabla v|^p dx : v \in W_0^{1,q}(\Omega) \quad \text{and} \int_\Omega n(x)|u|^q dx = 1\right\}. \end{aligned} \tag{1.2}$$

For $p = q = 2, m(x) = n(x) \equiv 1$, many results on the existence of weak solutions of the Problem (P) have been obtained by using the method of sub-super solutions and degree theory (see e.g. [1, 2]); For $p, q > 1, m(x), n(x) \in L^\infty(\Omega)$, Boccardo L and De Figueriredo D G[3] study the existence results of Problem (P) by means of Mountain Pass Lemma.

Our main tool here is the so-called fibering method introduced and developed by Pohozaev S I [4]. In 1997, Drabek P and Pohozaev S I [5] considered the existence of the solutions for a single equation of p-Laplacian by using fibering method; in 2003, Bozhkov Y and Mitidieri E [6] presented some existence and non-existence results of Problem (P) when $p, q > 1, m(x), n(x) \in L^\infty(\Omega)$ are essentially bounded functions.

It is interesting here that the functions $m(x), n(x)$ are just belonging to $L^r(\Omega), L^s(\Omega)$ respectively and may change sign. Our results will mainly rely on the results of Cuesta M [7], and consider the nonlinear eigenvalue problem

$$\begin{cases} -\Delta_p u \equiv -\mathrm{div}(|\nabla u|^{p-2}\nabla u) = \lambda m(x)|u|^{p-2}u, & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases} \tag{1.3}$$

where λ is the eigenvalue parameter, and $m(x)$ satisfies the condition (1.1).

Lemma 1.1 ([7]) *There exists a number $\lambda_1 > 0$ such that*

a) $\lambda_1 = \inf\left\{\int_\Omega |\nabla u|^p dx : u \in W_0^{1,p}(\Omega) \quad \text{and} \int_\Omega m(x)|u|^p dx = 1\right\}$ is the first positive eigenvalue of the problem (1.3);

b) the eigenfunctions associated to λ_1 are either positive or negative in Ω;

c) λ_1 is simple in the sense that the eigenfunctions associated to it are merely a constant multiple of each other;

d) λ_1 is isolated, that is, there exists $\delta > 0$ such that in the interval $(\lambda_1, \lambda_1 + \delta)$ there are no other eigenvalues of the problem (1.3).