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SHORT COMMUNICATION SECTION

# GLOBAL ATTRACTOR FOR MIXED INITIAL BOUNDARY VALUE PROBLEM FOR SOME MULTIDIMENSIONAL GINZBERG-LANDAU EQUATIONS

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**Abstract** The motivation of this paper is the study of the existence of weak global attractor for mixed initial boundary value problem for some multidimensional Ginzberg-Landau equations.

**Key Words** Nonlinear Ginzberg-Landau equations; global attractor; Galerkin method.

**2000 MR Subject Classification** 35Q35.

**Chinese Library Classification** O175.

The purpose of this is to investigate the existence of global attractor for mixed initial boundary value problem for some multidimensional Ginzberg-Landau equations

$$\overrightarrow{u_t} - \gamma \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial \overrightarrow{u}}{\partial x_j}\right) + b(x)q(|\overrightarrow{u}|^2)\overrightarrow{u} + c(x)\overrightarrow{u} = \overrightarrow{f}(x),\ x \in \Omega,\ t > 0, \quad (1)$$

$$\overrightarrow{u}(x,0) = \overrightarrow{u}_0(x), x \in \Omega, \quad (2)$$

$$\left.\left(\sum_{i,j=1}^{n} a_{ij}(x)\frac{\partial \overrightarrow{u}}{\partial x_i}\cos(\overrightarrow{n}, x_j) + h(x)\overrightarrow{u}\right)\right|_{\partial\Omega} = 0, \quad (3)$$

where $\overrightarrow{u} = (u_1(x,t), u_2(x,t), \cdots, u_N(x,t))$ is an unknown complex vector-value function, $\Omega$ is a bounded domain with boundary $\partial\Omega \in C^2$, $\overrightarrow{n}$ denotes the outward unit normal of $\partial\Omega$. On the complex functions $\overrightarrow{f}(x) = (f_1(x), f_2(x), \cdots, f_N(x))$, and the real function $a_{ij}(x)$, $c(x) = (c_{ij}(x))(i,j = 1,\cdots,N)$, $h(x)$, $b(x)$, $q(s)$, we make the following assumptions

(1) $\sum_{i,j=1}^{n} a_{ij}\xi_i\xi_j \geq a_0|\xi|^2$, $\sum c_{ij}\xi_i\xi_j \geq c_0|\xi|^2$, $\forall x \in \Omega$, $\xi = (\xi_1, \cdots, \xi_n) \in R^N$, $a_0 > 0$, $c_0 > 0$, $a_{ij} = a_{ji}$, $c_{ij} = c_{ji}$, $a_{ij}(x) \in C^1(\overline{\Omega})$.

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(2) $c_{ij}(x) \in L^\infty(Q_T)(i,j=1,\cdots,N)$, $Q_T=(0,T)\times\Omega$.

(3) $b(x)\geq 0$, $h(x)\geq 0$, $q(s)\geq 0$, $h(x)\in C^0(\overline{\Omega})$, $q(s)\in C^1(R^+)$, $b(x)\in C^0(\overline{\Omega})$.

(4) $\overrightarrow{f}(x)\in L^\infty(0,T;L^2(\Omega))$, $\overrightarrow{u}_0(x)\in H^2(\Omega)$, $\gamma=\gamma_0+i\gamma_1$, $\gamma_0>0$, $|\gamma|>0$.

By using the uniform estimates for $t$ we can get Theorem 1.

**Theorem 1** *Suppose that the problem (1)-(3) has a global smooth solution and the conditions of (1), (2), (3), (4) are satisfied; then there exists a global attractor $A$ of the initial-boundary value problem (1)-(3), i.e., there is a set $A$, such that*

*(i) $S_tA=A$, for $t\in R^+$.*

*(ii) $\lim\limits_{t\to\infty}\mathrm{dist}(S_tB,A)=0$, for any bounded set $B\subset H^2(\Omega)$, where*

$$\mathrm{dist}(S_tB,A)=\sup_{x\in B}\inf_{y\in A}\|x-y\|_E.$$

*and $S_t$ is a semi-group operator generator generated by the problem (1)-(3).*

**Proof** We know that there exists an operator semi-group generated by the problem (1)-(3). Thus we set the Banach space $E=H^2(\Omega)$, and $S_t:H^2(\Omega)\to H^2(\Omega)$. By using the results of Lemmas 1-4, and assuming that $B\subset H^2(\Omega)$ belongs to the ball $\{\|\overrightarrow{u}\|_{H^2}\leq R\}$, we have

$$\|S_t\overrightarrow{u}_0\|_E^2=\|\overrightarrow{u}(\cdot,t)\|_{H^2}^2\leq\|\overrightarrow{u}_0(x)\|_{H^2}^2+C_1\|\overrightarrow{f}(x)\|^2+C_2\leq R^2+C_3,(t\geq 0,u_0\in B).$$

where $C_1$, $C_2$, $C_3$ are absolute constants. This means that $\{S_t\}$ is uniformly bounded in $H^2$. Furthermore, from the results of the above Lemmas we see that

$$\|S_t\overrightarrow{u}_0\|_E^2=\|\overrightarrow{u}(\cdot,t)\|_{H^2}^2\leq 2(E_1+E_2+E_3+E_4),$$

$\forall t\geq t_0=T_0(R,\|\overrightarrow{u}_0\|_{H^2},\|\overrightarrow{f}(x)\|_{H^1})$, Hence

$$\overline{A}=\{\overrightarrow{u}(\cdot,t)\in H^2(\Omega),\|\overrightarrow{u}(\cdot,t)\|_{H^2(\Omega)}\leq 2(E_1+E_2+E_3+E_4)\},$$

is a bounded absorbing set of the operator semi-group $S_t$, thus we have the existence of weak compactness global attractor in $H^2$. The proof of the theorem is now completed.

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