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FORMATION OF SINGULARITIES FOR QUASILINEAR HYPERBOLIC SYSTEMS WITH CHARACTERISTICS WITH CONSTANT MULTIPLICITY

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Abstract In this paper we consider the Cauchy problem for quasilinear hyperbolic systems with characteristics with constant multiplicity. Without restriction on characteristics with constant multiplicity (>1), a blow-up result is obtained for the C^1 solution to the Cauchy problem under the assumptions where there is a simple genuinely nonlinear characteristic and the initial data possess certain weaker decaying properties.

Key Words Quasilinear hyperbolic systems; Cauchy problem; formation of singularity; life-span.

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1. Introduction and Main Result

Consider the following first order quasilinear hyperbolic system

$$\frac{\partial u}{\partial t}+A(u)\frac{\partial u}{\partial x}=0, \tag{1.1}$$

where $u=(u_1,\cdots,u_n)^T$ is the unknown vector function of (t,x) and $A(u)$ is an $n\times n$ matrix with suitably smooth elements $a_{ij}(u)$ $(i,j=1,\cdots,n)$.

By hyperbolicity, for any given u on the domain under consideration, $A(u)$ has n real eigenvalues $\lambda_1(u),\cdots,\lambda_n(u)$ and a complete set of left (resp. right) eigenvectors. Let $l_i(u)=(l_{i1}(u),\cdots,l_{in}(u))$ (resp. $r_i(u)=(r_{i1}(u),\cdots,r_{in}(u))^T$) be a left (resp. right) eigenvector corresponding to $\lambda_i(u)$ $(i=1,\cdots,n)$:

$$l_i(u)A(u)=\lambda_i(u)l_i(u)\quad (\text{resp. }\ A(u)r_i(u)=\lambda_i(u)r_i(u)). \tag{1.2}$$

We have

$$\det|l_{ij}(u)|\neq 0\quad (\text{resp.}\quad \det|r_{ij}(u)|\neq 0). \tag{1.3}$$

All $\lambda_i(u)$, $l_{ij}(u)$ and $r_{ij}(u)$ $(i,j=1,\cdots,n)$ are assumed to have the same regularity as $a_{ij}(u)$ $(i,j=1,\cdots,n)$.

Without loss of generality, we suppose that on the domain under consideration

$$l_i(u)r_j(u) \equiv \delta_{ij} \quad (i,j=1,\cdots,n) \tag{1.4}$$

and

$$r_i(u)^T r_i(u) \equiv 1 \qquad (i=1,\cdots,n), \tag{1.5}$$

where δ_{ij} stands for Kronecker's symbol.

In the case that the system (1.1) is strictly hyperbolic, namely, on the domain under consideration $A(u)$ has n distinct real eigenvalues

$$\lambda_1(u) < \lambda_2(u) < \cdots < \lambda_n(u),$$

F. John [1] and T.P. Liu [2] have obtained the blow-up phenomenon of C^2 solution to the Cauchy problem of the system (1.1) for small initial data with compact support, provided that all characteristics are genuinely nonlinear or a non-empty part of characteristics is genuinely nonlinear, while other part of characteristics is linearly degenerate. By introducing the concept of weak linear degeneracy, Li, Zhou and Kong [3-5] have given a complete result on the global existence and the blow-up phenomenon for the Cauchy problem of the system (1.1) with small and decaying C^1 initial data. Then, the results have been generalized to the non-strictly hyperbolic system with characteristics with constant multiplicity (see [6-7]), in which all characteristics with constant multiplicity (>1) are assumed to be linearly degenerate.

Recently, L.B. Wang [8-9] has considered the quasilinear hyperbolic system (1.1) with characteristics with constant multiplicity, in the case that there is no restriction on the linear degeneracy for characteristics with constant multiplicity (>1) or all characteristics with constant multiplicity (>1) are linearly degenerate and one single characteristic is genuinely nonlinear, respectively. In fact, for the Cauchy problem of the system (1.1) with the following initial data

$$t=0: \qquad u=\varepsilon\psi(x), \tag{1.6}$$

where $\varepsilon>0$ is a small parameter and $\psi(x)\in C^1$ satisfies

$$\sup_{x\in R}\Big\{(1+|x|)(|\psi(x)|+|\psi'(x)|)\Big\}<\infty$$

or

$$\sup_{x\in R}\Big\{(1+|x|)^{1+\widetilde{\mu}}|\psi'(x)|\Big\}<\infty, \tag{1.7}$$

where $\widetilde{\mu}>0$, she has respectively discussed the blow-up phenomenon of the C^1 solution to the Cauchy problem (1.1) and (1.6). Then, it is natural to propose the following question: what does happen when the initial data have weaker decaying rate? This paper is devoted to the study of this problem.