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ON THE RESTRICTED PARAMETER IDENTIFICATION PROBLEMS FOR GENERAL VARIATIONAL INEQUALITIES *

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Abstract

A restricted parameter identification problem for general variational inequalities is studied using the so-called asymptotic regularization method. Some applications, including the evolution dam problem, are also briefly discussed.

1. Introduction

In [4], Hoffmann and Sprekels discussed an identification problem of general variational inequalities under a functional analytic framework. The problem they treated is the following:

Let H, X be two Hilbert spaces, V be a separable and reflexive Banach space with dual V^* and X_0 be another Banach space, such that the embeddings $V \subset H \subset V^*$ are dense and continuous and X_0 is a dense subspace of X. The dual pairing between elements of V^* and V is denoted by $\langle \cdot , \cdot \rangle$. The inner product in X is denoted by $[\cdot , \cdot]$. Let $C \subset V$ be nonempty, closed and convex. Then, the identification problem is as follows:

Problem $(\hat{P})$. Given $u^* \in D(S) \cap C$ and $f^* \in V^*$, find $a^* \in X$, with $(a^*, u^*) \in D(A_2)$, such that there exists a $w^* \in S(u^*)$ which satisfies the variational inequality

$$\langle w^* + A_1(a^*) + A_2(a^*, u^*) - f^*, v - u^* \rangle + \Psi(v) - \Psi(u^*) \geq 0, \quad \forall v \in C \tag{1·1}$$

where, S, A_1, A_2, Ψ are some given operators which will be defined in the next section (or see [4]).

The main purpose of this paper is to discuss a similar problem with the restriction $a^* \in X$ replaced by $a^* \in K$ for some nonempty, closed and convex subset K of X_0. In many physical problems, the parameters we want to find should belong to some specific convex and closed set. The method we use is a combination of those used in [1], [4] and [6].

2. Solution to Finite Dimensional Problems

Let H, X, V, V^*, X_0 be the same as in section 1. The norms of these spaces are denoted by $\| \cdot \|_H$, $\| \cdot \|_X$, $\| \cdot \|_V$, $\| \cdot \|_{V^*}$ and $\| \cdot \|_{X_0}$, respectively. Let $C \subset V$ be nonempty, convex and closed. Let $K \subset X_0$ be nonempty, convex and closed in X. we consider the following restricted parameter identification problem:

Problem (P). Given $u^* \in D(S) \cap C, f^* \in V^*$ find $a^* \in K$, such that there

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$$\{T_n(a,u),(\hat a,\hat u)\} = \langle \frac{\varepsilon}{h}u + A_1(a) + A_2(a,u), \hat u\rangle + \left[\frac{a}{h}, \hat a\right] - \langle A_1(\hat a), u\rangle - \langle A_2(\hat a, u), u-u^*\rangle \quad (2\cdot 10)$$

$$g_n(a,u) = -\langle \frac{\varepsilon}{h}u_n + f^*, u\rangle + \Psi(u) - \left[\frac{a_n}{h}, a\right] + \langle A_1(a), u^*\rangle \quad (2\cdot 11)$$

For $w \in V^* \subset V_N^*$ we set

$$\{(0,w),(a,u)\} = \langle w, u\rangle, \ \forall (a,u) \in B \quad (2\cdot 12)$$

Thus, the $(n+1)$-th step of Problem (P_h) is equivalent to solve the following problem:

Problem (T_n). Find $(a,u) \in G$, with $u \in D(S)$, such that there exists $w \in S(u)$, satisfying

$$\{(0,w) + T_n(a,u), (\hat a,\hat u) - (a,u)\} + g_n(\hat a,\hat u) - g_n(a,u) \geq 0, \quad \forall (\hat a,\hat u) \in G \quad (2\cdot 13)$$

Theorem 2.1. *Suppose that* (A1) — (A7) *hold and* $\varepsilon > 0$ *is given. Then, there exists* $h_0 > 0$, *such that for any* $h \in (0, h_0]$, *Problem* (P_h) *has a solution* $\{(a_n,u_n)\}_{n=0}^{\infty}$.

Proof. It suffices to prove that for any $n \geq 1$, Problem (T_n) has a solution. To this end, like the proof of Theorem 2.1 of [4], we need to check several things.

First of all, since g_n is convex and continuous on B, we have

$$D(\partial g_n) = B \quad (2\cdot 14)$$

Let us define $Q: B \to 2^{B^*}$ by $Q(a,u) = \{0\} \times S(u)$. Then, Q is maximal monotone. By (A1) and (A6) and the definition of G, we have

$$(P_{W_M}(a^*), u^*) \in \mathrm{int}(D(Q)) \cap G \quad (2\cdot 15)$$

By (2.14), we have

$$\mathrm{int}(D(Q)) \cap D(\partial g_n) \neq \varnothing \quad (2\cdot 16)$$

and

$$G \cap \mathrm{int} D(Q + \partial g_n) = G \cap \mathrm{int} D(Q) \neq \varnothing \quad (2\cdot 17)$$

Also, the same argument as in [4], we have

$$\partial(g_n + X_G) = \partial g_n + \partial X_G \quad (2\cdot 18)$$

where

$$X_G(x) = \begin{cases} 0 & x \in G \\ +\infty & x \in G^C \end{cases} \quad (2\cdot 19)$$

Now, it remains to check T_n is continuous, bounded, coercive and pseudomonotone. Since $\dim W_M$, $\dim V_N < \infty$, we can assume that there exist $\alpha, \beta > 0$, such that

$$\| a \|_{X_0} \leq \alpha \| a \|_X, \quad \forall a \in W_M \quad (2\cdot 20)$$

$$\| u \|_V \leq \beta \| u \|_H, \quad \forall u \in V_N \quad (2\cdot 21)$$

It is easy to show that for any $(a,u), (\hat a,\hat u) \in B$

$$\| T_n(a,u) - T_n(\hat a,\hat u) \|_{B^*}^2 \leq \Big\{\Big(\frac{\varepsilon}{h} + \alpha\beta^2 \| A_2 \| \, \| \hat a \|_X\Big) \| u - \hat u \|_H + (\beta \| A_1 \| + \alpha\beta^2 \| A_2 \| \, \| u \|_H) \| a - \hat a \|_X\Big\}^2 + \Big\{[\beta \| A_1 \| + \alpha\beta^2 \| A_2 \| (\| u - u^* \|_H + \| u \|_H)] \cdot \| u - \hat u \|_H + \frac{1}{h} \| a - \hat a \|_X\Big\}^2 \quad (2\cdot 22)$$

which gives the continuity and boundedness of T_n. For the coercivity, let us take $(a,u) \in G$,

$$\{T_n(a,u),(a,u)\} = \langle \frac{\varepsilon}{h}u + A_1(a) + A_2(a,u), u\rangle + \frac{1}{h}\| a \|_X^2 - \langle A_1(a), u\rangle - \langle A_2(a,u), u - u^*\rangle$$

$$= \frac{\varepsilon}{h} \| u \|_H^2 + \frac{1}{h} \| a \|_X^2 + \langle A_2(a,u), u^*\rangle$$