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TWO TRANSFORMS ON LANDAU-LIFSHITZ EQUATIONS AND THEIR APPLICATION

Yang Ganshan
(Institute of Mathematics, Yunnan Normal University, Kunming 650091; Department of Mathematics, Yunnan Nationalities University, Kunming 650031, China)
(E-mail: ganshanyang@yahoo.com.cn)

To my teacher, Academician and Professor Guo Boling
for his 70th birthday.

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Abstract In this paper we obtain some new results on Landau-Lifshitz equation by two explicit transforms.

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1. Introduction

In this paper, we consider the following important type of Landau-Lifshitz equation

$$\frac{\partial u}{\partial t} = u \times (\triangle u + \lambda(t)H) + f(x,t,u) \tag{1.1}$$

where the spin vector $u = u(x,t) = u(x_1, x_2, \cdots, x_n; t) = (u_1, u_2, u_3)$ is a 3-dimensional vector valued unknown function with respect to space variables $x = (x_1, x_2, \cdots, x_n)$ and time t. The external magnetic field $\lambda(t)H$ is a 3-dimensional vector-valued function, where $\lambda(t)$ is only dependent on t and may be noncontinuous, even unbounded. "$\times$" denotes the cross-product of two 3-dimensional vectors. $f(x,t,u)$ are partial effective field that derives from some energy functional and are continuous with respect to x, t and u.

In 1935, the equation proposed by Landau and Lifshitz in [1] describes an evolution of spin fields in continuum ferromagnets, it bears a fundamental role in the understanding of non-equilibrium magnetism, just as the Navier-Stokes equation does in that of fluid dynamics.

On the case of one-dimensional motion, Nakamura and Sasada studied the following equation in 1974

$$\frac{\partial u}{\partial t} = u \times (\triangle u + H) \tag{1.2}$$

where the spin vector $u = u(x,t) = (u_1(x,t), u_2(x,t), u_3(x,t))$, $x \in R$, $H = (0,0,h)$. They found some solitary wave solutions (see [2]). Tjon and Wright also found some solitons under the case with non-vanishing external magnetic field in 1977 (see [3]). In 1976, Lakshmanan, Ruijgrok and Thompson constructed a class of solutions of (1.2) under the vanishing external magnetic field (i.e. $\lambda(t) \equiv 0$ or $H = 0$) in [4]. They found solitary waves have total energy localized in a finite region, with velocity of propagation inversely proportional to the width of this region.

On the higher dimensional case. In 1986, P.-L. Sulem, C. Sulem, C. Bardos proved that for any S_0 such that $|S_0(x)| = 1$ a.e. and $\partial S/\partial x_i \in L^2(R^d)$, there exists a global weak solution of $\partial S/\partial t = S \times \Delta S$, $S(x,0) = S_0(x)$ (see [5]). In the same year, Zhou and Guo in [6] proved the global existence of weak solution for generalized Landau-Lifhsitz equations without Gilbert term in multidimensional. They considered the homogeneous boundary problem

$$u(x,t) = 0, \quad \text{for } x \in \partial\Omega, \quad 0 \le t \le T \tag{1.3}$$

with the initial value condition

$$u(x,0) = \phi(x), \quad \text{for } x \in \Omega \tag{1.4}$$

for the system of ferromagnetic chain with several variables

$$u_t = u \times \Delta u + f(x,t,u) \tag{1.5}$$

where $f(x,t,u)$ is a given 3-dimensional vector function in $x \in R^n$, $t \in R^+$, $u \in R^3$. $\phi(x)$ is a given 3-dimensional initial value function on $\overline{\Omega}$, Ω is a bounded domain in n-dimensional Euclidean space R^n. Under some conditions on $f(x,t,u)$ and $\phi(x)$, they proved that the initial homogeneous boundary problem (1.3) and (1.4) with the system of ferromagnetic chain (1.5) has at least one global weak solution

$$u(x,t) \in L^\infty(0,T; H_0^1(\Omega)) \cap C^{(0,1/(3+[n/2]))}(0,T; L^2(\Omega)).$$

In 1992, F. Alouges, A. Soyeur established some necessary conditions for the existence of a global weak solution for the Landau-Lifshits equation with Gilbert term, which describe the evolution of spin fields in ferromagnetism: $\frac{\partial u}{\partial t} = u \times \Delta u - \lambda u \times (u \times \Delta u)$ (u taking values in R^3). They also established that, if λ is not equal to zero and u satisfies the Neumann boundary conditions, then there are infinitely many weak solutions (see [7]).

Whether there exists global smooth solution for multidimensional Landau-Lifshitz equation, ($n \ge 2$) is still an important open problem.