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IRROTATIONAL APPROXIMATION TO STEADY SUPERSONIC FLOW IN TWO SPACE VARIABLES*

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Abstract On the assumption that the total variation of the initial data is sufficiently small, we can use the stability results of Dafermos to get the L^2 estimate of the difference between the solutions to the isentropic steady Euler system and the potential flow equations with the same initial data.

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1. Introduction and the Main Results

The steady Euler system for a compressible isentropic fluid can be written as

$$\begin{cases}(\rho u)_x+(\rho v)_y=0,\\ (\rho u^2+p)_x+(\rho uv)_y=0,\\ (\rho uv)_x+(\rho v^2+p)_y=0,\end{cases}\tag{1.1}$$

where ρ is the density of the fluid, (u,v) is the velocity and the pressure $p=\rho^\gamma$ with $\gamma>1$.

The system has a simplified approximation:

$$\begin{cases}(\rho u)_x+(\rho v)_y=0,\\ v_x-u_y=0,\end{cases}\tag{1.2}$$

where the density and the velocity satisfy the following Bernoulli equation:

$$\frac{u^2+v^2}{2}+\frac{\gamma\rho^{\gamma-1}}{\gamma-1}=B,\tag{1.3}$$

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which gives $\rho = h(u,v)$ for $u^2+v^2<2B$:

$$h(u,v) = \frac{\gamma-1}{\gamma}(B-\frac{u^2+v^2}{2})^{\frac{1}{\gamma-1}}. \tag{1.4}$$

Here B is a constant.

The system (1.2) is called the potential flow equations. Denote $c=\sqrt{\gamma\rho^{\gamma-1}}$ which means the sound speed. When $u>c$, (1.1) and (1.2) are strictly hyperbolic. In [1–3], both the systems (1.1) and (1.2) are used to construct the global weak solutions for steady supersonic flows. The system (1.1) is an excellent model for the flow containing weak shocks since in approximates to the isentropic Euler system (1.2) up to the third order in shock strength. In [4], Zhang gives the L^1 estimate of the difference between the solutions to the system (1.1) and (1.2) with the same isentropic initial data. In this paper, the L^2 estimate of the difference will be discussed in a different way.

Let $U_0^{(0)}=(u_0,v_0)$ be a constant data satisfying the following conditions:

$$u_0 > c_0 = [(\gamma-1)B - \frac{\gamma-1}{2}(u_0^2+v_0^2)]^{\frac{1}{2}}, \tag{1.5}$$

$$u_0^2+v_0^2 < 2B \tag{1.6}$$

and $U_0^{(1)}=(u_0,v_0,h(u_0,v_0))$.

The main result is stated as follows:

Theorem 1.1 *Let $U_0^{(0)}=(u_0,v_0)$ be a constant data satisfying* (1.5) *and* (1.6), *and denoted by $U_0^*(y)=(u_0(y),v_0(y))$ be a bounded measurable function with small total variation such that $\lim\limits_{y\to\pm\infty} U_0^*(y)=U_0^{(0)}$. Let $U_1=(u_1,v_1,\rho_1)$ and $U_2=(u_2,v_2,h(u_2,v_2))$ be the solutions on $\mathbb{R}_x^+\times\mathbb{R}_y$ to Euler system* (1.1) *and system* (1.2) *taking $U_0 = (u_0(y),v_0(y),h(u_0(y),v_0(y)))$ as initial data respectively. Assume $T.V.(U_0^*)$ is sufficiently small, so that U_1 and U_2 are well defined for all $x>0$, then there exist constants $\delta>0$, $K>0$ such that for all U_0^* with $\|U_0^*-U_0^{(0)}\|_{L^\infty}+T.V.(U_0^*)<\delta$ and $U_0^*-U_0^{(0)}\in L^1$ and for $x>0$,*

$$\|U_1(x,\cdot)-U_2(x,\cdot)\|_{L^2(\mathbb{R})} \le Kx^{\frac{1}{2}}\|U_0^*-U_0^{(0)}\|_{BV}^{\frac{3}{2}}. \tag{1.7}$$

Here x is regarded as the time variable. $T.V.(U_0^*)$ stands for the total variation of $U_0^*(y)$, and $\|\cdot\|_{BV}=\|\cdot\|_{L^\infty}+T.V.(\cdot)$, while $\|\cdot\|_{L^\infty}$ is the L^∞ norm, $\|\cdot\|_{L^2}$ the L^2 norm.

Our approach to this problem is to compare both the weak solutions with the same regular solutions. The same idea has been used to treat the isentropic approximation for the full Euler system in one space dimension by Laure Saint Raymond in [5]. For the initial data are in the BV space, we first build smooth approximations to the given functions to obtain the local regular solutions. We will show that the lifespan of the local solutions can be determined by the supremum norm of the first order derivative