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# ESTIMATION FOR THE ASYMPTOTIC BEHAVIOR OF THE DELAYED COMPETITION MODEL*

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**Abstract** In ecological dynamic systems, the competition between species is a very universal phenomenon, which can be described by the well-known Volterra-Lotka model in a diffusion form. Noticing that the living space usually changes in a seasonal manner and the population development of the species may also undergo time-delay impact, a developed form of this model is investigated in this article. The main approaches employed here are the upper-lower solution method and the energy-estimate technique. The results show that whether the species may sustain survival or not depends on the relations among the birth rate, the death rate, the competition rate, the diffusivity and the time delay. For the survival case, the population evolutions of the two species may appear asymptotic periodicity with distinct upper bound and this bound depends heavily on the time delay. These results can be also checked by the intuitionistic numerical simulations.

**Key Words** Volterra-Lotka; diffusion; periodic; asymptotic; time delay.
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**Chinese Library Classification** O175.26, O174.29, Q141.

## 1. Introduction

In this paper, we mainly consider the delayed Volterra-Lotka competition diffusion model as follows:

$$L_1u_1(t,x) = u_1(t,x)[a(t,x) - b(t,x)u_1(t,x) - c(t,x)u_2(t-\tau_1,x)], \qquad (1.1)$$

$$L_2u_2(t,x) = u_2(t,x)[p(t,x) - q(t,x)u_2(t,x) - r(t,x)u_1(t-\tau_2,x)], \qquad (1.2)$$

$$B[u_i](t,x) = 0, \quad (t,x) \in R^+ \times \partial\Omega, \quad i = 1,2, \qquad (1.3)$$

$$u_i(t,x) = \phi_i(t,x), \quad (t,x) \in [-\tau_i,0] \times \overline{\Omega}, \quad i = 1,2. \qquad (1.4)$$

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Here $\Omega$ is a bounded domain in $R^n$ with smooth boundary $\partial\Omega$, $R^+ = (0, \infty)$, and the equations (1.1) and (1.2) are defined on $R^+ \times \Omega$. $L_i$ $(i = 1, 2)$ is an operator defined as $L_i = \partial/\partial t - d_i\triangle$, where $\triangle$ is the Laplacian, $d_1$ and $d_2$ are positive constants which denote the diffusive rates. The boundary operator $B$ is given by: $B[u](t, x) = u(t, x)$ (Dirichlet type) or $B[u](t, x) = \partial u(t, x)/\partial n$ (Neumann type) and $\partial/\partial n$ is the outward normal derivative on $\partial\Omega$. Here and in the follows we always assume that $u_1$ and $u_2$ have the same type of boundary condition. $\tau_i > 0$ $(i = 1, 2)$ denotes the time delay. $\phi_i \in C^{0,1}([-\tau_i, 0] \times \overline{\Omega})$ is a nonnegative bounded function which satisfies the compatibility condition: $B[\phi_i(0, x)] = 0$, $(\forall x \in \partial\Omega)$, $i = 1, 2$. The system (1.1)–(1.4) describes the evolution of two competing species $u_1$ and $u_2$ living in a bounded habitat $\Omega$. We are therefore only interested in the nonnegative solutions. Notice that the factors of the environment may vary spatially as well as temporally, and may also vary in a seasonal scale, it is quite natural to assume that the birth rates $a, p$, the self-limitation terms $b, q$ and the competition terms $c, r$ are periodic in time with period $T$. We also assume that they are Hölder continuous on $R^+ \times \overline{\Omega}$ with the bounds: $0 < a_1 \le a(t, x) \le a_2$; $0 < b_1 \le b(t, x) \le b_2$; $0 \le c_1 \le c(t, x) \le c_2$; $0 < p_1 \le p(t, x) \le p_2$; $0 < q_1 \le q(t, x) \le q_2$ and $0 \le r_1 \le r(t, x) \le r_2$ with $c(t, x) \not\equiv 0$ and $r(t, x) \not\equiv 0$.

For the study of this kind of model there are many researchers, such as in [1–3]. They have used the monotone methods and revealed the existence of periodic solutions of the boundary value problem. Notice that there is lack of uniqueness, we have proved it in [4]. Yet, to our knowledge, there are few results for it about the estimation on the asymptotic behavior. In our previous works [5-7], we have mentioned it and given out a method of estimation with respect to the Logistic model, the food-limited model and the Volterra model. In this paper we'll follow this method to reveal the effect of the time delay for the coexistence case. To this end, we first introduce some results about the no-delay boundary value problem:

$$\begin{cases} L_1\theta_1(t,x) = \theta_1(t,x)[a(t,x) - b(t,x)\theta_1(t,x) - c(t,x)\theta_2(t,x)], \\ L_2\theta_2(t,x) = \theta_2(t,x)[p(t,x) - q(t,x)\theta_2(t,x) - r(t,x)\theta_1(t,x)], \\ B[\theta_i](t,x) = 0, \quad (t,x) \in R^+ \times \partial\Omega, \quad i = 1,2, \end{cases} \tag{1.5}$$

here the first two equations are defined on $R^+ \times \Omega$. For the study of this boundary value problem there are many researchers, such as in [8–11]. Here we introduce the results given by [9, 10] for Neumann and [11] for Dirichlet boundary conditions, respectively. Denote $M = b/a$, $N = c/a$, $P = q/p$ and $Q = r/p$. Set $D_T = [0, T] \times \overline{\Omega}$ and for every bounded function $g : D_T \to R$, which stand for $M, N, P, Q$ etc., we define

$$\begin{cases} g_1 = \inf\{g(t,x) : (t,x) \in D_T\}, \qquad g_2 = \sup\{g(t,x) : x \in D_T\}, \\ \mathcal{N}(g) = \int_0^T \left(\min_{x\in\overline{\Omega}} g(t,x)\right) \mathrm{d}t, \qquad \mathcal{P}(g) = \int_0^T \left(\max_{x\in\overline{\Omega}} g(t,x)\right) \mathrm{d}t. \end{cases} \tag{1.6}$$