JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
Vol. **22**, No. 2, pp. 153-198

J. Partial Diff. Eq.
May 2009

Uniform Compact Attractors for a Nonlinear Non-Autonomous Equation of Viscoelasticity

QIN Yuming[1], DENG Shuxian[2,*] and SCHULZE Bert Wolfgang[3]

[1] *Department of Applied Mathematics, Donghua University, Shanghai 200051, China.*
[2] *College of Information Science and Technology, Donghua University, Shanghai 201620, China.*
[3] *Institut für Mathematik, Universität Potsdam, Postfach 60 15 53, 14415 Potsdam, Germany.*

Received 8 December 2007; Accepted 31 November 2008

Abstract. In this paper we prove the regularity, exponential stability of global solutions and existence of uniform compact attractors of semiprocesses, generated by the global solutions, of a two-parameter family of operators for a nonlinear one-dimensional non-autonomous equation of viscoelasticity. We employ the properties of the analytic semigroup to show the compactness for the semiprocess generated by the global solutions.

AMS Subject Classifications: 35B40, 35B41, 35B65, 35M20

Chinese Library Classifications: O175.27

Key Words: Exponential stability; absorbing set; C_0-semigroup; uniform compact attractor.

1 Introduction

In this paper we prove the regularity, exponential stability of global solutions and existence of uniform compact attractors of semiprocesses, generated by the global solutions, of a two-parameter family of operators for the following nonlinear one-dimensional non-autonomous viscoelasticity (see, e.g., [1–14])

$$y_{tt}=\sigma(y_x)y_{xx}+y_{xtx}+f(x,t), \qquad (x,t)\in(0,1)\times(\tau,+\infty), \tag{1.1}$$

$$y(0,t)=y(1,t)=0, \qquad t\geq\tau, \tag{1.2}$$

$$y(x,\tau)=y_0^\tau(x), \quad y_t(x,\tau)=y_1^\tau(x), \quad x\in(0,1), \quad \tau\in R^+=[0,+\infty), \tag{1.3}$$

*Corresponding author. *Email addresses:* `yuming_qin@hotmail.com` (Y. Qin), `dshuxian@163.com` (S. Deng), `schulze@math.uni-potsdam.de` (B. W. Schulze)

where $y=y^\tau(x,t)(\tau\in R^+)$ is an unknown function, σ is a real function defined on R, $f=f(x,t)$ is an external forcing term.

Since 1960's, the global well-posedness, asymptotic behavior of solutions and the investigation of the associated infinite-dimensional dynamical system have become the most essential aspects in the field of nonlinear evolution equations. For instance, the global well-posedness and large-time behavior of solutions to the problem (1.1)-(1.3) have attracted many mathematicians (see, e.g., [1–14] and references cited therein), and the development has seen great progress since then. Let us recall some of these achievements. When $\sigma(s)$ is smooth enough, $\sigma(s)>0$, $\forall s\in R$, $f\equiv 0$ (autonomous case), Greenberg et al. [4,6] the established existence, uniqueness and stability of smooth solution and Greenberg et al. [5] and Nishihara [10] proved the exponential decay of smooth solution. Yamada [13] weakened the regularity of initial data to obtain the global existence and exponential stability of smooth solution to the non-autonomous problem (1.1)-(1.3) (i.e., $f\not\equiv 0$). MacCamy [9] also obtained the existence, uniqueness and stability of smooth solutions to the more general equation:

$$y_{tt}=\sigma(y_x)y_{xx}+\lambda(y_x)y_{xtx}.$$

When $\sigma=\sigma(s)$ changes sign in $s\in R$, which corresponds to the model of phase transition, Andrew and Ball [15,16], and Pego [11] established the global existence, uniqueness and/or asymptotic behaviour of (weak) solutions for Eq. (1.1) with some boundary conditions. Kuttler and Hicks [7] proved the global existence and uniqueness of weak solutions for the more general equation:

$$y_{tt}=\sigma(y_x)_x+\Big(\alpha(y_x)y_{xt}\Big)_x+f$$

with some different boundary conditions from (1.2). Liu et al. [8] proved the global existence of solutions to initial boundary value problems or periodic boundary problem or Cauchy problem of the equation (1.1). Tsutsumi [12] obtained the global existence of solutions to initial boundary problem of (1.1). Yang and Song [14] investigated the blow-up results for initial boundary value problems of (1.1) with four types of boundary conditions and special versions of $\sigma(s)$.

As far as the associated infinite-dimensional dynamics is concerned, we refer to works [3,17–27] and references therein for related models. Generally speaking, for a given nonlinear evolution equation, once a global solution in all time $t>0$ has been established, a natural and interesting question is to ask the asymptotic behavior of the global solution as time t goes to infinity. The study of the asymptotic behavior of global solutions to nonlinear evolution equations as time goes to infinity can be divided into two categories. The first one is to investigate the asymptotic behavior of solution for any *given* initial datum. The second one is to investigate the asymptotic behavior of all global solutions when initial data vary *in bounded set*. The second category corresponds to the infinite-dimensional dynamics for nonlinear evolution equations. In this paper, we shall