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A Note to the Cauchy Problem for the Degenerate Parabolic Equations with Strongly Nonlinear Sources

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Abstract. In this note we study the nonexistence of nontrivial global solutions on $S=\mathbb{R}^N\times(0,\infty)$ for the following inequalities:

$$|u|_t\geq\Delta(|u|^{m-1}u)+|u|^q$$

and

$$|u|_t\geq\mathrm{div}(|\nabla u|^{p-2}\nabla|u|)+|u|^q.$$

When m,p,q satisfy some given conditions, the nonexistence of nontrivial global solution is proved, without taking their traces on the hyperplans $t=0$ into account.

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1 Introduction

In this note we study the nonexistence of nontrivial global solutions for the following inequality

$$|u|_t\geq\Delta(|u|^{m-1}u)+|u|^q,\qquad (x,t)\in S=\mathbb{R}^N\times(0,\infty),\tag{1.1}$$

where $m\geq1,\ q>1$. We prove that for any $q\in(m,m+\frac{2}{N}]$, the inequality (1.1) has no nontrivial solutions on S. As a simple consequence of this result, we obtain that for any $q\in(m,m+\frac{2}{N}]$ the inequality

$$u_t\geq\Delta(|u|^{m-1}u)+|u|^{q-1}u,\qquad (x,t)\in S=\mathbb{R}^N\times(0,\infty),\tag{1.2}$$

has no nontrivial nonnegative solutions on S. Although the result of the consequence is well-known (when $1<q<m+\frac{2}{N}$ see [1], when $q=m+\frac{2}{N}$ see [2]), the proof in this note is

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simpler and does not take the traces on the hyperplans $t=0$ into account. The case when $m=1$ is studied in [3].

Moreover, we also investigate the nonexistence of nontrivial global solutions for the following inequality

$$|u|_t \geq \mathrm{div}(|\nabla u|^{p-2}\nabla |u|)+|u|^q, \qquad (x,t)\in S=\mathbb{R}^N\times(0,\infty), \tag{1.3}$$

where $p>2$, $q>1$. We prove that for any $q\in(p-1,p-1+\frac{p}{N})$ the inequality (1.3) has no nontrivial solutions on S. As a simple consequence of this result, we obtain that for any $q\in(p-1,p-1+\frac{p}{N})$,

$$u_t \geq \mathrm{div}(|\nabla u|^{p-2}\nabla u)+|u|^{q-1}u, \qquad (x,t)\in S=\mathbb{R}^N\times(0,\infty), \tag{1.4}$$

has no nontrivial nonnegative global solutions on S. Although this result is proved in [4], again our proof is simpler and does not use the initial traces.

2 Main results

We will state the main results of this note; their (simpler) proofs will be provided in the following sections.

Definition 1. *$u\in L^q_{loc}(S)$ is call a solution of (1.1) if u satisfies*

$$\int_S(-|u|\varphi_t-|u|^{m-1}u\Delta\varphi)\mathrm{d}x\mathrm{d}t\geq\int_S|u|^q\varphi\mathrm{d}x\mathrm{d}t, \qquad \forall\varphi\in C_0^\infty(S),\ \varphi\geq 0. \tag{2.1}$$

Theorem 1. *Let $1\leq m<q\leq m+\frac{2}{N}$ and let u be a solution of (1.1) on S. Then $u(x,t)=0$ a.e. on S.*

Corollary 1. *Let $m\geq 1$, $m<q\leq m+\frac{2}{N}$. Then (1.2) has no nontrivial nonnegative global solution on S.*

The following definition and results are concerned with the inequalities (1.3) and (1.4).

Definition 2. *$u\in L^p_{loc}(0,\infty;W^p_{loc}(\mathbb{R}^N))\bigcap L^q_{loc}(S)$ is called a solution of (1.3) if u satisfies*

$$\int_S(-|u|\varphi_t+|\nabla u|^{p-2}\nabla|u|\nabla\varphi)\mathrm{d}x\mathrm{d}t\geq\int_S|u|^q\varphi\mathrm{d}x\mathrm{d}t, \qquad \forall\varphi\in C_0^\infty(S),\ \varphi\geq 0. \tag{2.2}$$

Theorem 2. *Let $p-1<q<p-1+\frac{p}{N}, p>2$ and let $u(x,t)$ be a solution of (1.3) on S. Then $u(x,t)=0$ a.e on S.*

Corollary 2. *Let $p-1<q<p-1+\frac{p}{N}, p>2$. Then (1.4) has no nontrivial nonnegative global solutions on S.*