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Asymptotic Behavior of Solution to Some Models Involving Two Species All with Chemotaxis

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Abstract. This paper is concerned with the asymptotic behavior of solution to the following model involving two species all with chemotaxis:

$$\begin{cases} \dfrac{\partial p}{\partial t}=D_p\nabla\Big(p\nabla\ln\dfrac{p}{w}\Big), \\ \dfrac{\partial q}{\partial t}=D_q\nabla\Big(q\nabla\ln\dfrac{q}{w}\Big), \\ \dfrac{\partial w}{\partial t}=\beta p-\delta w, \\ p\nabla\ln\Big(\dfrac{p}{w}\Big)\cdot\vec{n}=q\nabla\ln\Big(\dfrac{q}{w}\Big)\cdot\vec{n}=0. \end{cases}$$

We prove that the solution exists globally as $\beta\geq 0$. As $\beta<0$, whether the solution exists globally or not depends on the initial data. By function transformation and comparison, the asymptotical behavior of the solution is studied.

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Key Words: Chemotaxis model; asymptotical behavior of solution; blow-up; quenching; comparison.

1 Introduction

In 1970, Keller and Segel [1] introduced the classical chemotaxis equation as a model to describe the aggregation of slime mold amoebae Dicrocoelium Discoidal due to an attractive chemical substance:

$$\begin{cases} \dfrac{\partial u}{\partial t}=D_u\triangle u-\nabla(u\chi(w)\nabla w), \\ \dfrac{\partial w}{\partial t}=D_w\triangle w+hu-kw, \end{cases} \tag{1.1}$$

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where D_u and D_w are the diffusion coefficients of the cells u and attractant w, respectively, and $hu-kw$ is the kinetics term of w.

Such problems are derived from the chemotaxis movement which is a kind of response involving the detection of a chemical. In fact, all living system can sense the environment where they live and respond to it. The response usually involves movement towards or away from an external stimulus. The mechanism for such response is called taxis. The purpose of taxis range from movement toward food to avoidance of noxious substances to large-scale aggregation for survival. There are many different kinds of taxis, such as aerotaxis, chemotaxis, geotaxis and haptotaxis. Chemotaxis is a quite common phenomenon in bio-system. The term chemotaxis is used broadly in the mathematical literature to describe general chemosensitive movement responses. Chemotaxis can be either positive or negative. Models for chemotaxis have been applied to bacteria, slime molds, skin pigmentation patterns, leukocytes, etc. Many papers, see, e.g., [2–8], appear to demonstrate the role of chemotaxis using mathematical analysis or experimental stimulation.

However, not all chemotaxis phenomena can be described by the Keller-Segel models. Othmer and Stevens note that, in some cases, there is no diffusion for the chemical substance [9]. As a result, they establish the following Othmer-Stevens model (O-S model):

$$\begin{cases} \dfrac{\partial u}{\partial t}=D\dfrac{\partial}{\partial x}\Big(\dfrac{\partial u}{\partial x}-u\dfrac{\beta}{\alpha+\beta w}\dfrac{\partial w}{\partial x}\Big), & x\in(0,1),\ t>0,\\ \dfrac{\partial w}{\partial t}=F(u,w), & x\in(0,1),\ t>0,\\ u\dfrac{\partial}{\partial x}\ln\Big(\dfrac{u}{\alpha+\beta w}\Big)=0, & x=0,1,\\ u(x,0)=u_0(x)>0, & x\in(0,1),\\ w(x,0)=w_0(x)>0, & x\in(0,1). \end{cases} \tag{1.2}$$

The behavior of the solution to the above problem is studied in [9]. It is concluded that when w has linear growth, i.e.,

$$\frac{\partial w}{\partial t}=u-\mu w, \quad \mu>0, \quad \alpha>0,$$

the system has the constant solution $(u,w)=(u_0,u_0/\mu)$ which is asymptotically stable. If μ=0, then there is no time-independent solution, but there is a unstable space-independent solution $(u,w)=(u_0,w_0+u_0t)$ for any $\alpha\geq 0$ provided that $u(x,0)\equiv u_0,w(x,0)\equiv w_0$. However, when w has exponential growth

$$\frac{\partial w}{\partial t}=uw,$$

with boundary condition

$$u\frac{\partial}{\partial x}\ln\Big(\frac{u}{w}\Big)=0$$