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Asymptotically Self-Similar Global Solutions for a Higher-Order Semilinear Parabolic System

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Abstract. In this paper, we study the higher-order semilinear parabolic system

$$\begin{cases} u_t+(-\triangle)^m u=a|v|^{p-1}v, & (t,x)\in\mathbb{R}^1_+\times\mathbb{R}^N, \\ v_t+(-\triangle)^m v=b|u|^{q-1}u, & (t,x)\in\mathbb{R}^1_+\times\mathbb{R}^N, \\ u(0,x)=\varphi(x),\ v(0,x)=\psi(x), & x\in\mathbb{R}^N, \end{cases}$$

where $m,p,q>1$, $a,b\in\mathbb{R}$. We prove that the global existence of mild solutions for small initial data with respect to certain norms. Some of these solutions are proved to be asymptotically self-similar.

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Key Words: Higher-order parabolic equation; mild global solutions; asymptotically self-similar.

1 Introduction

This article is concerned with the Cauchy problem for the higher-order semilinear parabolic system

$$\begin{cases} u_t+(-\triangle)^m u=a|v|^{p-1}v, & (t,x)\in\mathbb{R}^1_+\times\mathbb{R}^N, \\ v_t+(-\triangle)^m v=b|u|^{q-1}u, & (t,x)\in\mathbb{R}^1_+\times\mathbb{R}^N, \\ u(0,x)=\varphi(x),\ v(0,x)=\psi(x), & x\in\mathbb{R}^N, \end{cases} \tag{1.1}$$

where $m,p,q>1$, $a,b\in\mathbb{R}$. Higher-order semilinear and quasilinear heat equations appear in numerous applications such as thin film theory, flame propagation, bi-stable phase

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transition and higher-order diffusion. We refer reader to the monograph [1] for some of these mathematical models. For studies of higher-order heat equations we refer also to [2–8] and the references therein.

Consider the following nonlinear parabolic equation:

$$\begin{cases} u_t+(-\triangle)^m u=|u|^p, & (t,x)\in\mathbb{R}^1_+\times\mathbb{R}^N,\\ u(0,x)=\varphi(x), & x\in\mathbb{R}^N. \end{cases}$$

It is well known that $p_F=1+2m/N$ is the critical Fujita exponent, see [2]. This model was later generalized to a weakly coupled system (1.1), where $a|v|^{p-1}v$ and $b|u|^{q-1}u$ are replaced by $|v|^p$ and $|u|^q$, respectively, see [4,5]. In [4], it was shown that for

$$\frac{N}{2m}>\max\Big\{\frac{1+p}{pq-1},\frac{1+q}{pq-1}\Big\},$$

there is a global solution with small initial data; while for

$$\frac{N}{2m}\le\max\Big\{\frac{1+p}{pq-1},\frac{1+q}{pq-1}\Big\}$$

and the positive energy on initial data, the solution will blow up in finite time. With the above condition, the life span of solution was studied in [5]. The technique used in [2] and [4] was based on a strongly continuous semigroup $S(t)$ generated by the infinitesimal generator $-(-\triangle)^m$. With it and the standard semigroup theory, problem (1.1) can be written as the following equivalent integral system:

$$\begin{cases} u(t)=S(t)\varphi+a\displaystyle\int_0^t S(t-\tau)(|v(\tau)|^{p-1}v(\tau))\mathrm{d}\tau,\\ v(t)=S(t)\psi+b\displaystyle\int_0^t S(t-\tau)(|u(\tau)|^{q-1}u(\tau))\mathrm{d}\tau, \end{cases} \tag{1.2}$$

where

$$S(t)\phi=b(t,\cdot)*\phi,\quad b(t,x)=t^{-N/(2m)}f(y),\quad y=x/t^{1/(2m)}. \tag{1.3}$$

In this paper, we shall prove the global existence of solutions for (1.1) with initial data $\Phi=(\varphi,\psi)$ small with respect to norm $\mathcal{N}$ (see (3.3)). In particular, if φ and ψ are homogeneous of degree $-2m(1+p)/(pq-1)$ and $-2m(1+q)/(pq-1)$ respectively, then the resulting solution of (1.1) is self-similar. In addition, we prove that the initial data of the form $(1-\eta)\varphi$ and $(1-\eta)\psi$, where η is a cut-off function, and φ and ψ are homogeneous of degree $-2m(1+p)/(pq-1)$ and $-2m(1+q)/(pq-1)$ respectively, give rise to asymptotically self-similar solutions. Our approach to self-similar solutions of (1.1) is based on the integral system (1.2) and contraction mapping. This is different from that exploited in previous works such as [6–8] on self-similar solutions, which are based on an analysis of the ordinary differential equation verified by the profile of the self-similar solution. The