JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **23**, No. 1, pp. 16-32

doi: 10.4208/jpde.v23.n1.2
February 2010

Remark on Random Attractors of Stochastic Non-Newtonian Fluid

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Received 6 November 2008; Accepted 23 August 2009

Abstract. In this paper, we study the asymptotic behaviors of solution for stochastic non-Newtonian fluid with white noise in two-dimensional domain. In particular, we will prove the existence of random attractors in H.

AMS Subject Classifications: 76A05, 60H15

Chinese Library Classifications: O175.29

Key Words: Random attractors; non-Newtonian fluid; stochastic equation; Ornstein-Uhlenbeck process.

1 Introduction

In this paper, we investigate the following stochastic incompressible non-Newtonian fluid in two-dimensional periodic domain D,

$$\mathrm{d}u+\Big(u\cdot\nabla u-\nabla\cdot\tau(e(u))+\nabla\pi\Big)\mathrm{d}t=f(x)\mathrm{d}t+\Phi\mathrm{d}W(t),\quad x\in D,\ t>0, \tag{1.1}$$

$$u(x,0)=u_0(x),\quad x\in D, \tag{1.2}$$

$$\nabla\cdot u(x,t)=0, \tag{1.3}$$

subject to the periodic boundary conditions

$$u(x,t)=u(x+L\chi_j,t),\quad \int_D u(x,t)\mathrm{d}x=0,\quad D=[0,L]^2\ \ (L>0), \tag{1.4}$$

where $\{\chi_j\}_{j=1}^2$ is the natural basis of R^2.

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The unknown vector function u denotes the velocity of the fluid, f is the external force function, and the scalar function π represents the pressure, $\tau_{ij}(e(u))$ is a symmetric stress tensor. There are many fluid materials such as liquid foams, polymeric fluids such as oil in water, blood, etc. whose viscous stress tensors are represented by the form

$$\tau_{ij}(e(u))=2\mu_0\left(\epsilon+|e(u)|^2\right)^{\frac{p-2}{2}}e_{ij}(u)-2\mu_1\Delta e_{ij}(u),\quad i,j=1,2,\ \epsilon>0,\ p>2, \tag{1.5}$$

$$e_{ij}(u)=\frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\Big),\quad |e(u)|^2=\sum_{i,j=1}^{2}|e_{ij}(u)|^2.$$

We use

$$W(t)=\sum_i \beta_i(t)h_i \tag{1.6}$$

to describe the cylindrical Wiener process for white noise on Hilbert space H adapted to a filtration $(\mathscr{F}_t)_{t\in R}$ on a fixed probability space $(\Omega,\mathscr{F},\mathbb{P})$, where $\{h_i\}$ is an orthonormal complete basis in Hilbert space H and $\beta_i(t)$ is a family of mutually independent real valued standard Wiener process. Φ is a predictable process in a fixed probability space, which is also a linear mapping.

Next, we set some notations. $L^q(D)$ denotes the Lebesgue space with norm $\|\cdot\|_{L^p}$, particularly, $\|\cdot\|_{L^2}=\|\cdot\|$, and $\|u\|_{L^\infty}=\text{esssup}_{x\in D}|u(x)|$. $H^\sigma(D)$ represents the Sobolev space $\{u\in L^2(D),\ D^k u\in L^2(D),\ k\le\sigma\}$, with $\|\cdot\|_{H^\sigma}=\|\cdot\|_\sigma$. $\mathscr{C}(I,X)$ denotes the space of continuous functions from the interval I to X. $L^q(0,T;X)$ is the space of all measurable functions $u:[0,T]\mapsto X$, with the norm

$$\|u\|^q_{L^q(0,T;X)}=\int_0^T\|u(t)\|_X^q\mathrm{d}t,$$

and when $q=\infty$,

$$\|u\|_{L^\infty(0,T;X)}=\text{ess}\sup_{t\in[0,T]}\|u(t)\|_X.$$

Define a space of smooth functions that incorporates the periodicity with respect to x and divergence-free condition

$$\mathscr{V}=\left\{u\in C^\infty_{per}(D):\nabla\cdot u=0,\int_D u\mathrm{d}x=0\right\}.$$

We use H to denote the closure of $\mathscr{V}$ in $L^2(D)$ with norm $\|\cdot\|$; $\dot{H}^\sigma(D)$ the closure of $\mathscr{V}$ in $H^\sigma(D)$ with norm $\|\cdot\|_\sigma$ $(\sigma\ge 1)$. Particularly, when $\sigma=2$, $V=\dot{H}^2(D)$. Denote by $(\dot{L}_2^{0,\sigma},\|\cdot\|_{\dot{L}_2^{0,\sigma}})$ the Hilbert space of Hilbert-Schmidt operators from H to $\dot{H}^\sigma(D)$, with the norm

$$\|\Phi\|_{\dot{L}_2^{0,\sigma}}=\left(\sum_i\|\Phi h_i\|^2_{\dot{H}^\sigma}\right)^{\frac{1}{2}}. \tag{1.7}$$

A final restriction on Φ is given: Φ belongs to $\dot{L}_2^{0,5}$.