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On the Non-Trivial Solvability of Boundary Value Problems in the Angle Domains

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Abstract. In the first part of the present paper we deal with the first boundary value problem for general second-order differential equation in plane angle. The criterion of non-trivial solvability is obtained for such problem in space C^2 of functions having polynomial growth at infinity. In the second part so-called "almost Cauchy" problem in a polygon for high order differential equation without respect of type is investigated. The necessary condition of uniqueness violation of solution is appeared to be sufficient in case of problem with one boundary condition.

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1 Introduction

General boundary value problems for elliptic equations in domains with angle points have been studied by Eskin in [1], where he has proved the normal solvability of such problems in plane domain with boundary data, satisfying Lopatinsky condition in C^N with $N>0$ large enough. First boundary value problem has been investigated by Fufayev [2], Volkov [3]. Besides, in [4] Kondratiev considered general boundary value problem for elliptic equation in domain, whose boundary contains the finite number of conic points. In this connection, solution was examined in special spaces of functions having derivatives summable with some weight. Author proves, that solution is smooth everywhere and that, in general, derivatives have power singularities on approaching to the conic point.

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In the first part of the present paper we treat the uniqueness violation of solution of the first boundary value problem in an angle, posed for the second order differential equation with heterogeneous symbol and complex coefficients. It should be noted, that the equation under consideration is irrespective of type. We find a necessary and sufficient condition under which the Dirichlet problem has a non-trivial solution in spaces of temperate growth functions for some class of differential equations, concerned with given angle. Besides, we compare the result with well-known solution of the Laplace equation in angle.

To remove the principal difficulty connected with the absence of local analyticity of the Fourier transform acting on the solution extension to the whole plane, we make a symbol shift on vector λ belonging to the tubular domain $T^C \in \mathbb{C}^2$ and thus obtain the differential operator with homogeneous symbol. Then we apply the duality method [5] to the derived problem. It is significant, that the tubular domain mentioned above depends on angle.

In the second part of the present paper we research the "almost Cauchy" problem in a polygon for high order differential equation without respect of type. The number of boundary conditions is less on unit than the order of the differential operator. The necessary condition of uniqueness violation of solution is appeared to be sufficient when we deal with a problem having one boundary condition.

2 Statement of problem

Let Ω be plane angle of measure α with generatrices $\widetilde{b}^1$ and $\widetilde{b}^2$ and vertex, located at zero, and let $b^1=(b^1_1,b^1_2)$, $b^2=(b^2_1,b^2_2)$ be normal vectors with respect to straight lines $(b^1\cdot x)=0$, $(b^2\cdot x)=0$. We consider the homogeneous Dirichlet problem

$$u|_{\partial\Omega}=0, \tag{2.1}$$

for second-order differential equation

$$\widetilde{L}u=L\left(\frac{\partial}{\partial x}+\lambda\right)u=\Big(a^1\cdot(\nabla+\lambda)\Big)\Big(a^2\cdot(\nabla+\lambda)\Big)u=0 \tag{2.2}$$

in angle Ω (see Fig. 1), where a^1, a^2, $\lambda\in\mathbb{C}^2$ are complex constant vectors. Moreover, a^1, a^2 are arbitrary, but the choice of complex vector $\lambda\in T^C\subset\mathbb{C}^2$ is restricted by set T^C, depending on angle.

As to conditions at infinity, the solution is assumed to belong to the Schwartz space S'. We also note, that the heterogeneous symbol $\widetilde{l}(\xi)$ of the differential operator $\widetilde{L}(\frac{\partial}{\partial x})$ becomes the homogeneous one after shift on λ:

$$\widetilde{l}(\xi-\lambda)=l(\eta)=a\eta_1^2+b\eta_1\eta_2+c\eta_2^2.$$