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Global Strong Solutions for the Viscous, Micropolar, Compressible Flow

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Abstract. In this paper, we consider the viscous, micropolar, compressible flow in one dimension. We give the proof of existence and uniqueness of strong solutions for the initial boundary problem that vacuum can be allowed initially.

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1 Introduction

In this paper, we consider the micropolar fluids, describing the viscous, compressible fluids with randomly oriented particles suspended in the medium when the deformation of fluid particles is ignored, which is first introduced by E. Eringen [1]. The system is governed by the following equations:

$$\varrho_t+(\varrho u)_x=0, \tag{1.1}$$

$$(\varrho u)_t+(\varrho u^2)_x+P_x(\varrho)=\lambda u_{xx}, \tag{1.2}$$

$$(\varrho w)_t+(\varrho uw)_x+\mu w=\nu w_{xx}. \tag{1.3}$$

Here the unknown functions ϱ, u, and w represent the density, the velocity and microrotational velocity, respectively. The pressure $P(\varrho)=A\varrho^\gamma$, with the adiabatic exponent $\gamma\geq 1$ and the gas constant $A>0$. The positive constants λ, μ, and ν are the viscosities.

We study the initial boundary value problem of (1.1)–(1.3) in a bounded spatial domain, without loss of generality, we consider in $I=(0,1)$. We prescribe the initial conditions

$$(\varrho,u,w)(0,x)=(\varrho_0,u_0,w_0)(x), \quad x\in I, \tag{1.4}$$

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and the no-slip boundary conditions for the velocity and microrotational velocity

$$(u,w)|_{x=0,1}=0. \tag{1.5}$$

There have been a lot of studies on viscous, compressible, micropolar fluids, we refer the readers to [2–5] and references therein. [2–5] prove the global existence and regularity of the solutions, it depends on the initial density ϱ_0 has a positive lower bound. The important point here is that we allow the initial density may vanish in an open subset. Our main theorem is following:

Theorem 1.1. *Assume that* $\varrho_0\geq 0$, $\varrho_0\in H_0^1(I)$, $(u_0,w_0)\in H_0^1(I)\cap H^2(I)$, *and the following compatibility conditions*

$$\lambda u_{0xx}-(A\varrho_0^\gamma)_x=\varrho_0^{1/2}f, \qquad \text{for some} f\in L^2(I), \tag{1.6}$$
$$\nu w_{0xx}-\mu w_0=\varrho_0^{1/2}g, \qquad \text{for some} g\in L^2(I). \tag{1.7}$$

Then there exists a unique global strong solution (ϱ,u,w) *to the initial boundary value problem* (1.1)–(1.5) *for all* $T\in(0,+\infty)$,

$$\begin{aligned}&\varrho\in L^\infty(0,T;H^1(I)),\quad (u,w)\in L^\infty(0,T;H_0^1(I)\cap H^2(I)),\\&(\varrho_t,\sqrt{\varrho}u_t,\sqrt{\varrho}w_t)\in L^\infty(0,T;L^2(I)),\ \ (u_t,w_t)\in L^2(0,T;H^1(I)).\end{aligned}$$

2 Proof of Theorem 1.1

This section we give the proof of Theorem 1.1. Firstly, we derive a priori estimates on (ϱ,u,w). In this section we denote C the various generic positive constants depending only on the initial data and T.

The first estimate is easy to get from the energy inequality.

Lemma 2.1.

$$\sup_{t\in[0,T]}(\|\sqrt{\varrho}u\|_{L^2}+\|\sqrt{\varrho}w\|_{L^2}+\|G(\varrho)\|_{L^1})+\int_0^T(\|u_x\|_{L^2}+\|w_x\|_{L^2})\mathrm{d}t\leq C,$$

where G *is the nonnegative function defined by*

$$G(\varrho)=\begin{cases}A\varrho^\gamma/(\gamma-1), & \text{if } \gamma>1,\\ A[(1+\varrho)\ln(1+\varrho)-\varrho], & \text{if } \gamma=1.\end{cases}$$

Lemma 2.2.

$$\sup_{t\in[0,T]}\|\varrho\|_{L^\infty}\leq C.$$