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Two Regularity Criteria Via the Logarithm of the Weak Solutions to the Micropolar Fluid Equations

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Abstract. In this note, a logarithmic improved regularity criteria for the micropolar fluid equations are established in terms of the velocity field or the pressure in the homogeneous Besov space.

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1 Introduction

In this paper, we consider the following Cauchy problem for the incompressible micropolar fluid equations :

$$\begin{cases} \partial_t u+(u\cdot\nabla)u-\Delta u+\nabla\pi-\nabla\times\omega=0, \\ \partial_t\omega-\Delta\omega-\nabla\mathrm{div}\omega+2\omega+u\cdot\nabla\omega-\nabla\times u=0, \\ \nabla\cdot u=0, \\ u(x,0)=u_0(x),\ \omega(x,0)=\omega_0(x), \end{cases} \tag{1.1}$$

where $u=u(x,t)\in\mathbb{R}^3$, $\omega=\omega(x,t)\in\mathbb{R}^3$ and $\pi=\pi(x,t)$ denote the unknown velocity vector field, the micro-rotational velocity and the unknown scalar pressure of the fluid at the point $(x,t)\in\mathbb{R}^3\times(0,T)$, respectively, while u_0, ω_0 are given initial data with $\nabla\cdot u=0$ in the sense of distributions.

The global regularity of the weak solution in the 3D case is still a big open problem. Therefore it is interesting problem on the regularity criterion of the weak solutions under

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assumption of certain growth conditions on the velocity or on the pressure. As for the velocity regularity, Dong and Chen [1] (see also [2]) proved the regularity of weak solutions under the velocity condition

$$\nabla u \in L^q(0,T;\dot{B}^0_{p,r}(\mathbb{R}^3)), \qquad \frac{2}{q}+\frac{3}{p}=2, \qquad \frac{3}{2}<p\leq\infty, \qquad r\leq\frac{2p}{3}.$$

As for the pressure criterion, Yuan [3] studied the regularity of weak solutions in Lorentz spaces

$$\pi\in L^q(0,T;L^{p,\infty}(\mathbb{R}^3)), \qquad \text{for } \frac{2}{q}+\frac{3}{p}=2,\ \frac{3}{2}<p<\infty$$

or

$$\nabla\pi\in L^q(0,T;L^{p,\infty}(\mathbb{R}^3)), \qquad \text{for } \frac{2}{q}+\frac{3}{p}=3,\ 1<p<\infty.$$

Zhang et al [4] recently improved the regularity from Lorentz to Besov spaces

$$\pi\in L^q(0,T;B^r_{p,\infty}(\mathbb{R}^3)), \qquad \frac{2}{q}+\frac{3}{p}=2+r, \qquad \frac{3}{2+r}<p<\infty, \qquad -1<r\leq 1.$$

The aim of the present study is to investigate Logarithmically improved regularity criterion for the micropolar fluid equations in terms of the gradient of velocity and pressure in Besov spaces.

2 Preliminaries and main result

We recall the definition and some properties of the space we are going to use.

Definition 2.1 ([5]). *Let $\{\varphi_j\}_{j\in\mathbb{Z}}$ be the Littlewood-Paley dyadic decomposition of unity that satisfies $\widehat{\varphi}\in C_0^\infty(B_2\backslash B_{1/2})$, $\widehat{\varphi}_j(\xi)=\widehat{\varphi}(2^{-j}\xi)$ and $\sum_{j\in\mathbb{Z}}\widehat{\varphi}_j(\xi)=1$ for any $\xi\neq 0$, where B_R is the ball in $\mathbb{R}^3$ centered at the origin with radius $R>0$. The homogeneous Besov space is defined by $\dot{B}^s_{p,q}=\{f\in\mathcal{S}'/\mathcal{P}: \|f\|_{\dot{B}^s_{p,q}}<\infty\}$ with norm*

$$\|f\|_{\dot{B}^s_{p,q}}=\left(\sum_{j\in\mathbb{Z}}\left\|2^{js}\varphi_j*f\right\|^q_{L^p}\right)^{\frac{1}{q}}$$

for $s\in\mathbb{R}$, $1\leq p,q\leq\infty$, where $\mathcal{S}'$ is the space of tempered distributions and $\mathcal{P}$ is the space of polynomials.

It is easy to see the inequality

$$\|f\|_{\dot{B}^0_{\infty,\infty}}\leq C\|f\|_{BMO}\leq C\|f\|_{\dot{B}^0_{\infty,2}}$$

holds for $f\in BMO$, where BMO is the space of the bounded mean oscillations.