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Global Existence and Uniqueness of Solution for a Class of Non-Newtonian Fluids

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Abstract. In this paper, we consider a class of non-Newtonian fluids in one-dimensional bounded interval. The global existence and uniqueness of solution are investigated.

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1 Introduction

In this paper, we consider a class of compressible non-Newtonian fluids in one-dimensional bounded intervals:

$$\begin{cases} \rho_t + (\rho v)_x = 0, & (x,t)\in\Omega_T, \\ (\rho u)_t + (\rho v u)_x - \left(|u_x|^{p-2}u_x\right)_x + \pi_x = \rho f, & (x,t)\in\Omega_T, \\ \pi = A\rho^\gamma, & A>0,\ \gamma>1, \end{cases} \tag{1.1}$$

with the initial and boundary conditions

$$\begin{cases} (\rho,u)|_{t=0} = (\rho_0,u_0), & x\in[0,1], \\ u|_{x=0} = u|_{x=1} = 0, & t\in[0,T], \end{cases} \tag{1.2}$$

where ρ, u and π denote the unknown density, velocity and pressure, respectively. The motion of the fluid is driven by an external force f, v is a given function, the initial density $\rho_0 \ge \underline{\rho} > 0$, $\Omega_T = I\times(0,T), I=(0,1)$, $p>2$, where $\underline{\rho}$ is a positive constant.

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In the recent three decades, fluid dynamics has attracted the attention of many mathematicians and engineers. The Navier-Stokes equations are generally accepted as right governing equations for the compressible or motion of viscous fluid, which is usually described by the principle of conservations of mass and momentum, and we deduce finally

$$\begin{cases} \rho_t+\mathrm{div}(\rho u)=0, \\ (\rho u)_t+\mathrm{div}(\rho u\otimes u)-\mathrm{div}(\Gamma)+\nabla\pi=\rho f, \end{cases} \tag{1.3}$$

where Γ denotes the viscous stress tensor, which depends on $E_{ij}(\nabla u)$, and

$$E_{ij}(\nabla u)=\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}$$

is the rate of strain. If the relation between the stress tensor Γ_{ij} and the rate of strain E_{ij} is linear, then the fluid is called Newtonian. That is, Newtonian fluids satisfy the following linear relation

$$\Gamma=\mu E_{ij}(\nabla u).$$

The coefficient of proportionality μ is called the viscosity coefficient, and it is a characteristic material quantity for the fluid concerned, which in general depends on density, temperature and pressure. Generally speaking, air, other gases, water, motor oil, alcohols, simple hydrocarbon compounds and others tend to be Newtonian fluids. The governing equations of motions of them can be described by Navier-Stokes equations. If the relation between the stress tensor Γ_{ij} and the rate of stain E_{ij} is nonlinear, then the fluid is called to be non-Newtonian. For instance, molten plastics, polymer solutions and paints tend to be non-Newtonian fluids. The simplest model of the stress-strain relation for such fluids is given by the power laws, which states that

$$\Gamma=\mu\left|\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right|^q,$$

for $0<q<1$ (see, e.g., [1]).

Ladyzhenskaya proposed in [2] a special form for Γ on the incompressible model:

$$\Gamma_{ij}=\big(\mu_0+\mu_1|E_{ij}(\nabla u)|^{p-2}\big)E_{ij}(\nabla u).$$

These models are called

$$\begin{cases} \text{Newtonian} & \text{for } \mu_0>0,\ \mu_1=0, \\ \text{Rabinowitsch} & \text{for } \mu_0,\ \mu_1>0,\ p=4, \\ \text{Eills} & \text{for } \mu_0,\ \mu_1>0,\ p>2, \\ \text{Ostwald-de Waele} & \text{for } \mu_0=0,\ \mu_1>0,\ p>1, \\ \text{Bingham} & \text{for } \mu_0,\ \mu_1>0,\ p=1. \end{cases}$$