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# Nonexistence of Blow-Up Flows for Symplectic and Lagrangian Mean Curvature Flows

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**Abstract.** In this paper we mainly study the relation between $|A|^2$, $|H|^2$ and $\cos\alpha$ ($\alpha$ is the Kähler angle) of the blow up flow around the type II singularities of a symplectic mean curvature flow. We also study similar property of an almost calibrated Lagrangian mean curvature flow. We show the nonexistence of type II blow-up flows for a symplectic mean curvature flow satisfying $|A|^2 \leq \lambda|H|^2$ and $\cos\alpha \geq \delta > 1-\frac{1}{2\lambda}$ ($\frac{1}{2} \leq \lambda \leq 2$), or for an almost calibrated Lagrangian mean curvature flow satisfying $|A|^2 \leq \lambda|H|^2$ and $\cos\theta \geq \delta > \max\{0, 1-\frac{1}{\lambda}\}$ ($\frac{3}{4} \leq \lambda \leq 2$), where $\theta$ is the Lagrangian angle.

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**Key Words**: Symplectic surface; Lagrangian surface; mean curvature flow.

## 1 Introduction

Suppose that $M$ is a compact Kähler surface, and $\Sigma$ is an oriented closed surface smoothly immersed in $M$. Let $\omega$ be the Kähler form on $M$. The Kähler angle [1] $\alpha$ is defined by

$$\omega|_\Sigma = \cos\alpha d\mu_\Sigma, \tag{1.1}$$

where $d\mu_\Sigma$ is the area element of $\Sigma$ of the induced metric from $M$. We say that $\Sigma$ is a holomorphic curve if $\cos\alpha \equiv 1$, $\Sigma$ is a Lagrangian surface if $\cos\alpha \equiv 0$ and $\Sigma$ is a symplectic surface if $\cos\alpha > 0$.

There are many results on symplectic mean curvature flows.

It is proved in [2] that if the scalar curvature of the Kähler-Einstein surface is positive and the initial surface is sufficiently close to a holomorphic curve, the mean curvature flow has a global solution and it converges to a holomorphic curve.

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In general, the mean curvature flow may develop singularities as time evolves. Chen-Li [3] and Wang [4] proved the nonexistence of type I singularities along the symplectic mean curvature flow. However, there may have type II singularities.

One of the most important examples of type II singularity is the translating soliton (c.f. [5, 6]). Han-Li [7] and Han-Sun [8] cleared out the translating solitons to the symplectic mean curvature flow under certain conditions. In [9], Han-Li-Sun showed the nonexistence of type II blow-up flow of a symplectic mean curvature flow which is normal flat.

Suppose that $M$ is a compact Calabi-Yau complex surface with a Kähler form $\omega$, a complex structure $J$ and a parallel holomorphic (2,0) form $\Omega$. Let $\Sigma$ be a Lagrangian surface in $M$, we have (see [10])

$$\Omega|_{\Sigma}=e^{i\theta}d\mu_{\Sigma},$$

where $\theta$ is a multivalued function and is well-defined up to an additive constant $2k\pi$, $k\in\mathbb{Z}$. We call $\theta$ the Lagrangian angle. We say $\Sigma$ is spectial if $\theta\equiv$constant, and $\Sigma$ is almost calibrated if $\cos\theta>0$.

The almost calibrated Lagrangian mean curvature flows share many properties with the symplectic mean curvature flow (c.f. [11–17]).

In this paper, we mainly study the relation between $|A|^2$, $|H|^2$ and $\cos\alpha$ of the blow up flow around the type II singularities. We consider a general mean curvature flow $\Sigma_t$ in $\mathbb{R}^4$ which exists globally with bounded second fundamental forms and the following property:

$$\mu_t(\Sigma_t\cap B_R(0))\leq CR^2, \tag{1.2}$$

where $0<C<\infty$ is a constant independent of $t$ and $R$.

We follow some ideas in [9] to prove the non-existence of type II blow-up flows for a symplectic mean curvature flow which satisfies

$$|A|^2\leq\lambda|H|^2 \tag{1.3}$$

and $\cos\alpha\geq\delta>1-\frac{1}{2\lambda}$ $(1/2\leq\lambda\leq 2)$. Analogously we have a corresponding result for the almost calibrated Lagrangian mean curvature flow. The main theorems are as follows.

**Theorem 1.1.** *Suppose that $\Sigma_t$, $t\in(-\infty,0]$ is a complete symplectic mean curvature flow with*

$$\cos\alpha\geq\delta>1-\frac{1}{2\lambda}\quad(1/2\leq\lambda\leq 2) \tag{1.4}$$

*in $\mathbb{C}^2$ which satisfies* (1.2). *If*

$$\sup_{t\in(-\infty,0]}\sup_{\Sigma_t}|A|^2=1, \tag{1.5}$$

*then there exist $x_0,t_0$ such that*

$$|A|^2(x_0,t_0)>\lambda|H|^2(x_0,t_0). \tag{1.6}$$