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Some Exact Solutions of Two Fifth Order KdV-Type Nonlinear Partial Differential Equations

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Abstract. We consider the generalized integrable fifth order nonlinear Korteweg-de Vries (fKdV) equation. The extended Tanh method has been used rigorously, by computational program MAPLE, for solving this fifth order nonlinear partial differential equation. The general solutions of the fKdV equation are formed considering an ansatz of the solution in terms of tanh. Then, in particular, some exact solutions are found for the two fifth order KdV-type equations given by the Caudrey-Dodd-Gibbon equation and the another fifth order equation. The obtained solutions include solitary wave solution for both the two equations.

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1 Introduction

The nonlinear partial differential equations are visible in a wide variety of physical problems in the field of fluid dynamics, plasma physics, solid mechanics and quantum field theory. This is also noticed to arise in engineering, chemical and biological applications. The application of nonlinear traveling waves has been brought prosperity in the field of applied science.

It is noted that various special methods such as auto-Backlund transformation, inverse scattering theory, Riccati method, tanh-coth method, sech method, sine-cosine method and so on are using to handle higher order nonlinear wave equations. All these methods can be applied to the nonlinear KdV equation and to the KdV-type equations for finding special kind of solutions.

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The generalized fifth order Korteweg-de Vries (fKdV) equation is given [1,2] by

$$u_t+\alpha u^2u_x+\beta u_xu_{xx}+\gamma uu_{xxx}+u_{xxxxx}=0, \tag{1.1}$$

where, u is a function of the spatial variable x and the time variable t . The coefficients α,β,γ are arbitrary nonzero and real constants.

Eq. (1.1) is an important mathematical model that has a wide application in quantum mechanics and nonlinear optics. This equation is integrable and is invariant under the scaling transformation:

$$x\to e^{\epsilon}x, \quad t\to e^{\mu\epsilon}t, \quad u\to e^{-\lambda\epsilon}u, \tag{1.2}$$

with $\epsilon>0$ and λ,μ any scaling constants.

For different values of α,β,γ, Eq. (1.1) represents different fifth order KdV-type evolutionary equations. For $\alpha=180,\beta=30,\gamma=30$, it gives a 5th order KdV-type equation which is known as Caudrey-Dodd-Gibbon (CDG) equation [3,4]. The CDG equation is completely integrable and invariant under above scaling transformation. This solitary wave equation was formally derived by Caudrey, Dodd and Gibbon in 1976. Again for $\alpha=180$, $\beta=45$, $\gamma=30$, it represents another 5^{th} order KdV-type equation [5] which is also integrable. Both these two equations are associated with scattering problems and admitted Miura type transformation [6].

The tanh method is well-known for solving one dimensional nonlinear wave and evolution equations. This method gives more closed form traveling wave solution of the nonlinear wave equations. We use extended tanh method, by MAPLE, to find the general solution of the fifth order KdV-type nonlinear partial differential equation (1.1). Then we show that the fKdV equation has real solution if parameters α,β,γ satisfy the relation either

$$40\alpha\le(\beta+2\gamma)^2 \qquad \text{or} \qquad \alpha=\frac{1}{10}(\beta+\gamma)\gamma. \tag{1.3}$$

This is interesting to find the traveling wave solution of the other 5^{th} order integrable partial differential equations using the general solutions of fKdV equation under above conditions. In particular examples, we consider the Caudrey-Dodd-Gibbon equation and the another 5^{th} order KdV-type equation to obtain some exact solutions. The obtained solutions include bell-shaped sech^2 solitary wave solutions of these two equations. We have described that the solitary wave solution carries the conserved quantities which are constant of motion and that the solution has a decaying behavior. We draw this bell-shaped solitary wave figure for the CDG equation with two different speeds.

2 The extended Tanh method

The Tanh method [7] was first introduced by Malfliet and Hereman in 1976 as a reliable treatment of some nonlinear equations. This method describes that if we take a traveling wave variable

$$\xi=x-ct, \tag{2.1}$$