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Dynamics for Controlled 2D Generalized MHD Systems with Distributed Controls

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Abstract. We study the dynamics of a piecewise (in time) distributed optimal control problem for Generalized MHD equations which model velocity tracking coupled to magnetic field over time. The long-time behavior of solutions for an optimal distributed control problem associated with the Generalized MHD equations is studied. First, a quasi-optimal solution for the Generalized MHD equations is constructed; this quasi-optimal solution possesses the decay (in time) properties. Then, some preliminary estimates for the long-time behavior of all solutions of Generalized MHD equations are derived. Next, the existence of a solution of optimal control problem is proved also optimality system is derived. Finally, the long-time decay properties for the optimal solutions is established.

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1 Introduction

The control of viscous flows is very crucial to many technological and scientific applications. We are motivated to study the asymptotic behaviors and dynamics of solutions for the controlled Generalized MHD equations (GMHD). In this paper we study the long time behavior of the solution for optimal control problems associated with GMHD equations on the infinite time interval. The optimal control with the systems governed by Navier-Stokes and Boussinesq equations was studying by L. Hou and Y. Yan [1] and by H. Chun Lee and B. Chun Shin [2], respectively. This work is motivated by the desire to steer over time a candidate velocity field u and magnetics field b to a target velocity field

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U and magnetics field B by appropriately controlling the body force. The existence of solutions of GMHD equations was studied in [3].

We formulate here a controllability problem for the GMHD equations: find a triplet (u,b,f) such that the functional

$$J_{(0;+\infty)}(u,b,f)=\frac{\alpha}{2}\int_0^{+\infty}\int_\Omega |(u,b)-(U,B)|^2 \mathrm{d}x\mathrm{d}t+\frac{\beta}{2}\int_0^{+\infty}\int_\Omega |f-F|^2 \mathrm{d}x\mathrm{d}t, \tag{1.1}$$

is minimized subject to the 2-D GMHD equations:

$$\frac{\partial u}{\partial t}+(u\cdot\triangledown)u-(b\cdot\triangledown)b+\nabla p+\nu(-\triangle)^r u=f, \qquad \text{in } \Omega\times(0,\infty), \tag{1.2}$$
$$\frac{\partial b}{\partial t}+(u\cdot\triangledown)b-(b\cdot\triangledown)u+\theta(-\triangle)^r b=0, \qquad \text{in } \Omega\times(0,\infty), \tag{1.3}$$
$$\triangledown\cdot u=0,\ \ \triangledown\cdot b=0, \qquad \text{in } \Omega\times(0,\infty), \tag{1.4}$$
$$u=0, \triangle u=0,\ \ \triangle^2 u=0,\ \cdots,\ \triangle^r u=0, \qquad \text{on } \partial\Omega\times(0,\infty), \tag{1.5}$$
$$b\cdot\mathbf{n}=0,\ \ \triangle b=0,\ \ \triangle^2 b=0,\ \cdots,\ \triangle^r b=0, \qquad \text{on } \partial\Omega\times(0,\infty), \tag{1.6}$$
$$\nabla u=0,\ \ \nabla^2 u=0,\ \cdots,\ \nabla^r u=0, \qquad \text{on } \partial\Omega\times(0,\infty), \tag{1.7}$$
$$\nabla b=0,\ \ \nabla^2 b=0,\ \cdots,\ \nabla^r b=0, \qquad \text{on } \partial\Omega\times(0,\infty), \tag{1.8}$$
$$u(0)=u_0 \quad \text{and} \quad b(0)=b_0, \tag{1.9}$$

where $\mathbf{n}$ is an outward unit normal vector on $\partial\Omega$ and r a non negative integer, also $\nu>0$ and $\theta>0$ are the kinematic viscosity and conductivity parameters, respectively. Here $\alpha,\beta>0$ are given constants, Ω is a bounded, sufficiently smooth domain in $\mathbb{R}^2$ with $\partial\Omega$ denoting its boundary; U, B and F are a given desired velocity field, a given desired magnetic field and a given desired body force, respectively. Also, f is a distributed control (body force), and u,b and p denote the velocity field, the magnetic field and the pressure field, respectively.

S. S. Ravindran [4] was interesting in the standard case $(r=1)$ and he had also used a curl term. The periodic case was studying by M. Gunzburger and C. Trenchea in [5] and [6].

Intuitively, if a flow field (u,b) is close to the desired field (U,B), then the body force corresponding to the two fields (u,b) and (U,B) should also be close. Hence, in order that the optimal control solution of GMHD equations is close to the desired field (U,B), we must place some restrictions on the desired body force F involved in the cost functional (1.1). In fact, throughout this paper we will simply choose for some $P\in L_0^2(\Omega)$,

$$F:=\partial_t U+\nu(-\triangle)^r U+(U.\nabla)U+\nabla P-(B.\nabla)B \tag{1.10}$$

for the desired field (U,B) satisfying

$$\partial_t B+\theta(-\triangle)^r B+(U.\triangledown)B-(B.\triangledown)U=0. \tag{1.11}$$