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Existence of a Renormalised Solutions for a Class of Nonlinear Degenerated Parabolic Problems with L^1 Data

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Abstract. We study the existence of renormalized solutions for a class of nonlinear degenerated parabolic problem. The Carathéodory function satisfying the coercivity condition, the growth condition and only the large monotonicity. The data belongs to $L^1(Q)$.

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1 Introduction

Let Ω be a bounded open set of $\mathbb{R}^N$, p be a real number such that $2\leq p<\infty$, $Q=\Omega\times]0,T[$ and $w=\{w_i(x),\ 0\leq i\leq N\}$ be a vector of weight functions (i.e., every component $w_i(x)$ is a measurable function which is positive a.e. in Ω) satisfying some integrability conditions. The objective of this paper is to study the following problem in the weighted Sobolev space:

$$\begin{cases}\dfrac{\partial b(u)}{\partial t}-\mathrm{div}(a(x,t,u,Du))+\mathrm{div}(\phi(u))=f, & \text{in } Q,\\ b(x,u)(t=0)=b(x,u_0), & \text{in } \Omega,\\ u=0, & \text{on } \partial\Omega\times]0,T[.\end{cases} \tag{1.1}$$

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The function b is assumed to be a strictly increasing C^1-function, the data f and $b(u_0)$ lie in $L^1(Q)$ and $L^1(\Omega)$, respectively. The functions ϕ is just assumed to be continuous of $\mathbb{R}$ with values in $\mathbb{R}^N$, and the Carathéodory function a satisfying only the large monotonicity (see assumption (H_2)).

Let us point out, the difficulties that arise in problem (1.1) are due to the following facts: the data f and u_0 only belong to L^1, a satisfies the large monotonicity that is

$$[a(x,t,s,\xi)-a(x,t,s,\eta)](\xi-\eta)\geq 0, \quad \text{for all } (\xi,\eta)\in\mathbb{R}^N\times\mathbb{R}^N,$$

and the function $\phi(u)$ does not belong to $(L^1_{loc}(Q))^N$ (because the function ϕ is just assumed to be continuous on $\mathbb{R}$). To overcome this difficulty, we will apply Landes's technical (see [1,2]) and the framework of renormalized solutions. This notion was introduced by Diperna and P.-L. Lions [3] in their study of the Boltzmann equation. This notion was then adapted to an elliptic version of (1.1) by L. Boccardo et al. [4] when the right hand side is in $W^{-1,p'}(\Omega)$, by J.-M. Rakotoson [5] when the right hand side is in $L^1(\Omega)$, and finally by G. Dal Maso, F. Murat, L. Orsina and A. Prignet [6] for the case of right hand side is general measure data.

For the parabolic equation (1.1) the existence of weak solution has been proved by J.-M. Rakotoson [7] with the strict monotonicity and a measure data, the existence and uniqueness of a renormalized solution has been proved by D. Blanchard and F. Murat [8] in the case where $a(x,t,s,\xi)$ is independent of s, $\phi=0$, and by D.Blanchard, F. Murat and H. Redwane [9] with the large monotonicity on a.

For the degenerated parabolic equations the existence of weak solutions have been proved by L. Aharouch et al. [10] in the case where a is strictly monotone, $\phi=0$ and $f\in L^{p'}(0,T,W^{-1,p'}(\Omega,w^*))$. See also the existence of renormalized solution by Y.Akdim et al [11] in the case where $a(x,t,s,\xi)$ is independent of s and $\phi=0$.

Note that, this paper can be seen as a generalization of [9,10] in weighted case and as a continuation of [11].

The plan of the paper is as follows. In Section 2 we give some preliminaries and the definition of weighted Sobolev spaces. In Section 3 we make precise all the assumptions on a, ϕ, f and u_0. In Section 4 we give some technical results. In Section 5 we give the definition of a renormalized solution of (1.1) and we establish the existence of such a solution (Theorem 5.1). Section 6 is devoted to an example which illustrates our abstract result.

2 Preliminaries

Let Ω be a bounded open set of $\mathbb{R}^N$, p be a real number such that $1<p<\infty$ and $w=\{w_i(x), 0\leq i\leq N\}$ be a vector of weight functions, i.e., every component $w_i(x)$ is a measurable function which is strictly positive a.e. in Ω. Further, we suppose in all our considerations