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Existence and Asymptotic Behavior of Boundary Blow-Up Weak Solutions for Problems Involving the p-Laplacian

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Abstract. Let $D\subset\mathbb{R}^N(N\geq 3)$, be a smooth bounded domain with smooth boundary ∂D. In this paper, the existence of boundary blow-up weak solutions for the quasilinear elliptic equation $\Delta_p u=\lambda k(x)f(u)$ in $D(\lambda>0$ and $1<p<N)$, is obtained under new conditions on k. We give also asymptotic behavior near the boundary of such solutions.

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1 Introduction

Let $D\subset\mathbb{R}^N(N\geq 3)$, be a smooth bounded domain with smooth boundary ∂D. In this work, we consider the boundary blow-up elliptic problem

$$\begin{cases} \Delta_p u=\lambda k(x)f(u) & \text{in } D, \\ u=\infty & \text{on } \partial D, \end{cases} \tag{1.1}$$

where $\Delta_p u$ is the p-Laplacian $\Delta_p u:=\operatorname{div}(|\nabla u|^{p-2}\nabla u)$, with $1<p<N$ and $\lambda>0$. The boundary condition is understood as $u(x)\to\infty$ as $\delta(x)\to 0$, where $\delta(x)$ denotes the Euclidian distance between x and ∂D. The solutions to the above problem are called boundary blow-up (explosive-large) solutions. The nonlinearity f is assumed to fulfill either

(**F1**) $f\in C^1(\mathbb{R}); f'\geq 0$ on $\mathbb{R}$ and $f\geq 0$, on $[0,\infty)$,

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or

($\mathbf{F'1}$) $f\in C^1([0,\infty));f'\geq 0$ on $[0,\infty)$ and $f\geq 0$ on $[0,\infty)$,

and the following Keller-Osserman condition:

(**K.O**) f is a single-value real continuous function satisfying

$$\int_a^\infty \Big[F(x)\Big]^{\frac{-1}{p}}dx<\infty, \text{ for some } a>0, \text{ where } F(x)=\int_0^x f(s)ds.$$

We assume throughout this paper that the function k satisfies the following conditions:

(**K0**) $k\in L^q_{loc}(D)$ for some $q>p^{*\prime}$ and the problem

$$\begin{cases} \Delta_p u=-k(x) & \text{in } D,\\ u=0 & \text{on } \partial D, \end{cases} \tag{1.2}$$

has a weak solution $h\in W^{1,p}_{loc}(D)\cap C_0(\overline{D})$, (see Definition 3.1 for the definition of weak solution.)

(**K1**) $k\geq 0$ on D and for every $x_0\in D$ there exist a constant $\delta_{x_0}>0$ and a domain D_{x_0} such that $x\in D_{x_0}\subset D$ and $k\geq\delta_{x_0}$ on a neighborhood of ∂D_{x_0}.

When $\lambda=1$ the problem (1.1) becomes

$$\begin{cases} \Delta_p u=k(x)f(u) & \text{in } D,\\ u=\infty & \text{on } \partial D. \end{cases} \tag{1.3}$$

Boundary blow-up solutions to the problem (1.3) have been extensively studied when $p=2$, we quote the pioneering works [1–9] and the reference therein. When the weight k is bounded, problem (1.3) has been considered by several authors [10–12].

In the reference situation $k\equiv 1$, Diaz and Letelier [13] proved the existence of solution to (1.3) when $f(t)=t^q$, $q>p-1$. Matero [14] established the existence of solutions to (1.3) when f satisfies (**F′1**) and (**K.O**). He also gave asymptotic behavior of the solutions near the boundary.

Recently, the existence of solutions to (1.3), with k nonnegative and continuous on D, was studied in some works under the following assumptions on f:

(f1) $f\in C^1(0,\infty)$, $f'\geq 0$, $f(0)=0$ and $f>0$ on $(0,\infty)$,

(f2) $\displaystyle\int_1^\infty f^{\frac{-1}{p-1}}(t)dt=\infty$.

In [15], Wu and Yang considered the case when $k(x)=a(r)\in C[0,1)$ is nonnegative and nontrivial. They proved that problem (1.3) has a solution on $B(0,1)$ if and only if

$$\lim_{r\to 1}\int_0^r\Big(s^{1-N}\int_0^t t^{N-1}a(t)dt\Big)^{\frac{1}{p-1}}ds=\infty.$$