JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **26**, No. 4, pp. 373-384

doi: 10.4208/jpde.v26.n4.5
December 2013

Many Families of Exact Solutions for Nonlinear System of Partial Differential Equations Describing the Dynamics of DNA

ZAYED Elsayed M. E.[1,*] and ARNOUS Ahmed H.[2]

[1] *Mathematics Department, Faculty of Science, Zagazig University, Zagazig, Egypt.*
[2] *Department of Engineering Mathematics and Physics, Higher Institute of Engineering, El Shorouk, Egypt.*

Received 2 May 2013; Accepted 29 June 2013

Abstract. The objective of this paper is to apply the improved generalized Riccati equation mapping method to find many families of exact traveling wave solutions for the general nonlinear dynamic system in a new double-chain model of DNA. This model consists of two long elastic homogeneous strands connected with each other by an elastic membrane. Hyperbolic and trigonometric function solutions of this model are obtained. Comparison between our results and the well-known results are given.

AMS Subject Classifications: 35K99, 35P99, 35P05

Chinese Library Classifications: O177.7, O177.9

Key Words: Nonlinear dynamics of DNA model; exact solutions; improved generalized Riccati equation mapping method; double-chain model of DNA.

1 Introduction

The investigation of the traveling wave solutions for nonlinear partial differential equations plays on important role in the study of nonlinear physical phenomena. Nonlinear wave phenomena appears in various scientific and engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics and geochemistry. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations. Because of the increased concentration in the theory of solitary waves, a large variety of

*Corresponding author. *Email addresses:* e.m.e.zayed@hotmail.com (E. M. E. Zayed), ahmed.h.arnous@gmail.com (A. H. Arnous)

analytic and computational methods have been established in the analysis of the nonlinear models (see [1–18]). An attractive nonlinear model for the nonlinear science is the deoxyribonucleic acid (DNA). The dynamics of DNA molecules is one of the most fascinating problems of modern biophysics because it is at the basis of life. The DNA structure has been studied during last decades. The investigation of DNA dynamics has successfully predicted the appearance of important nonlinear structures. It has been shown that the nonlinearity is responsible for forming localized waves. These localized waves are interesting because they have the capability to transport energy without dissipation [19–30]. In [27,30], it is given that a new double-chain model of DNA consists of two long elastic homogeneous strands which represent two polynucleotide chains of the DNA molecule, connected with each other by an elastic membrane representing the hydrogen bonds between the base pair of the two chains. Under some appropriate approximation, the new double-chain model of DNA can be described by the following two general nonlinear dynamical equations:

$$u_{tt}-c_1^2u_{xx}=\lambda_1 u+\gamma_1 uv+\mu_1 u^3+\beta_1 uv^2, \tag{1.1}$$
$$v_{tt}-c_2^2v_{xx}=\lambda_2 v+\gamma_2 u^2+\mu_2 u^2v+\beta_2 v^3+c_0, \tag{1.2}$$

where

$$c_1=\pm\sqrt{\frac{Y}{\rho}};\quad c_2=\pm\sqrt{\frac{F}{\rho}}; \tag{1.3a}$$
$$\lambda_1=\frac{-2\mu}{\rho\sigma h}(h-l_0);\quad \lambda_2=\frac{-2\mu}{\rho\sigma}; \tag{1.3b}$$
$$\gamma_1=2\gamma_2=\frac{2\sqrt{2}\mu l_0}{\rho\sigma h^2};\quad \mu_1=\mu_2=\frac{-2\mu l_0}{\rho\sigma h^3}; \tag{1.3c}$$
$$\beta_1=\beta_2=\frac{4\mu l_0}{\rho\sigma h^3};\quad c_0=\frac{\sqrt{2}\mu(h-l_0)}{\rho\sigma}, \tag{1.3d}$$

where ρ,σ,Y and F denote respectively the mass density, the area of transverse cross-section, the Young's modulus and the tension density of each strand; μ is the rigidity of the elastic membrane; h is the distance between the two strands, and l_0 is the height of the membrane in the equilibrium position. In Eqs. (1.1) and (1.2), u is the difference of the longitudinal displacements of the bottom and top strands, while v is the difference of the transverse displacements of the bottom and top strands.

The objective of this article is to construct many families of exact solutions of the system (1.1) and (1.2) using the improved generalized Riccati equation mapping method. The rest of this article is organized as follows: In Section 2, the description of the improved generalized Riccati equation mapping method is given. In Section 3, we apply this method to find many families of exact traveling wave solutions of the system (1.1) and (1.2). In Section 4, some conclusions and discussions are given.