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Wavelet Collocation Methods for Viscosity Solutions to Swing Options in Natural Gas Storage

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Abstract. This paper presents the wavelet collocation methods for the numerical approximation of swing options for natural gas storage in a mean reverting market. The model is characterized by the Hamilton-Jacobi-Bellman (HJB) equations which only have the viscosity solution due to the irregularity of the swing option. The differential operator is formulated exactly and efficiently in the second generation interpolating wavelet setting. The convergence and stability of the numerical scheme are studied in the framework of viscosity solution theory. Numerical experiments demonstrate the accuracy and computational efficiency of the methods.

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1 Introduction

The aim of this paper is to investigate the application of adaptive wavelet collocation methods for Hamilton-Jacobi-Bellman (HJB) equations arising from pricing swing options in a mean reverting market.

Models of swing options are an extension of the Black-Scholes model. Due to the uncertainty of future consumption and the limited fungibility of many commodities, some commodity markets have introduced swing options which give the consumer flexibility with respect to both the timing and the amount of commodity delivered. For descriptions of swing options, we refer to [1,2] and the references therein. Swing options are very common in energy markets, because they provide consumers with flexibility to vary their rate

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of consumption without being exposed to price fluctuations, which can be extreme, especially in the case of electricity. For swing options on electricity, see [3]; on gas, see [4]; on coal, see [5], for example.

Due to their importance in the energy markets, the pricing of swing options has gained more and more attention over the last decade, and much effort has been expanded in designing algorithms for pricing swing options. The discrete valuation of swing options has been studied by several authors. In [1], a discrete forest methodology is developed for swing options as a dynamically coupled system of European options. Also in [2], a binomial/trinomial forest is built to calculate the price of swing options. In [6] and [7], Monte Carlo techniques are employed for pricing swing options. Continuous time models allow the use of powerful mathematical tools to analyze the properties of solutions and have recently appeared in the literature. A continuous time model for the price of the general commodity-based swing option is presented in [8], where the price function is the solution of a system of quasi-variational inequalities. In [9], a continuous time model is built for pricing swing options on natural gas in a mean reverting market, where the price function is the solution of a HJB equation.

The more powerful the model is, the more important it is to develop the right computational tools to get reliable information out from the model. In this paper, we study the numerical solution of swing option models presented in [9, 10], where a finite-element approach is developed to solve this class of models. Furthermore, the stochastic meshes are applied in [11] and the optimal exercise boundary estimation is applied in [12] respectively for solving swing option models. For further survey about swing options, we refer the readers to [13].

Since optimization strategies are involved in swing options, in regions where the optimal exercise strategy is a rapidly-changing function of the price, the solution may exhibit less regularity, which will be problematic for nonadaptive (uniform grid) methods. Therefore, we develop wavelet-based methods for pricing swing options. This framework allows for using finer resolution where needed and coarser resolution in smooth areas, and thereby improves the approximation efficiency.

This paper is organized as follows. In Section 2, we introduce the efficient formulation of operators in a wavelet collocation setting. In Section 3, we briefly introduce the swing option models to be studied in this paper. In Section 4, we present a wavelet-based numerical scheme to the proposed HJB system. In Section 5, the convergence analysis is performed in the framework of viscosity solution theory. In Section 6, the numerical results are presented. Conclusions are drawn in Section 7.

2 Second generation interpolating wavelets

2.1 Scaling functions on an interval

Consider the interval $\Omega=[0,1]$. For each level j, we place a grid