JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **27**, No. 4, pp. 347-356

doi: 10.4208/jpde.v27.n4.5
December 2014

Blow-up of Solutions for a Class of Fourth-order Equation Involving Dissipative Boundary Condition and Positive Initial Energy

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Received 18 October 2014; Accepted 25 December 2014

Abstract. In this paper we consider a forth order nonlinear wave equation with dissipative boundary condition. We show that there are solutions under some conditions on initial data which blow up in finite time with positive initial energy.

AMS Subject Classifications: 35B30; 35B44; 35G31

Chinese Library Classifications: O175.27

Key Words: Blow up; fourth order; boundary dissipation.

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1 Introduction

In this article, we are concerned with the problem

$$u_{tt}+\Delta[(a_0+a|\Delta u|^{m-2})\Delta u]-b\Delta u_t=g(x,t,u,\Delta u)+|u|^{p-2}u,\quad x\in\Omega,\quad t>0, \tag{1.1}$$

$$u(x,t)=0,\quad \Delta u(x,t)=-c_0\partial_\nu u(x,t),\quad x\in\partial\Omega,\quad t>0, \tag{1.2}$$

$$u(x,0)=u_0(x),\quad u_t(x,0)=u_1(x),\quad x\in\Omega, \tag{1.3}$$

where $\Omega\subset R^n$ is a bounded domain with smooth boundary $\partial\Omega$ in order that the divergence theorem can be applied. ν is the unit normal vector pointing toward the exterior of Ω and $p>m+1>3$. Moreover, the constants a_0,a,b,c_0 are positive numbers and $g(x,t,u,\Delta u)$ is a real function that satisfies specific condition that will be enunciated later.

The one-dimension case of the fourth order wave equation is written as

$$u_{tt}+u_{xxxx}-a(u_x^2)_x=f(x),\quad x\in\Omega\subset R,\quad t>0, \tag{1.4}$$

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which was first introduced in [1] to describe the elasto-plastic-microstructure models for the longitudinal motion of an elasto-plastic bar. Chen and Yang [2] studied the Cauchy problem for the more general Eq. (1.4).

Young Zhou, in [3] studied the following nonlinear wave equation with damping and source term on the whole space:

$$u_{tt}+a|u_t|^{m-1}u_t-\phi\Delta u=f(x,u),$$

where $a,b>0,m\geq 1$ are constants and $\phi(x):R^N\to R,n\geq 2$. He has obtained the criteria to guarantee blow up of solutions with positive initial energy, both for linear and nonlinear damping cases.

In [4], the same author has been studied the following Cauchy problem

$$\begin{aligned}&u_{tt}+au_t-\Delta u=b|u|^{p-1}u, \quad x\in R^N,t>0,\\&u(x,0)=u_0(x),\ u_t(x,0)=u_1(x), \quad x\in R^N,\end{aligned}$$

where $a,b>0$. He proved that the solution blows up in finite time even for vanishing initial energy if the initial datum (u_0,u_1) satisfies $\int_{R^N}u_0u_1\mathrm{d}x\geq 0$. (See also [5])

Recently, in [6] Bilgin and Kalantarov investigated blow up of solutions for the following initial-boundary value problem

$$\begin{aligned}&u_{tt}-\nabla[(a_0+a|\nabla u|^{m-2})\nabla u]-b\Delta u_t=g(x,t,u,\nabla u)+|u|^{p-2}u, \quad x\in\Omega, \quad t>0,\\&u(x,t)=0, \quad x\in\partial\Omega, \quad t>0,\\&u(x,0)=u_0(x), \quad u_t(x,0)=u_1(x), \quad x\in\Omega.\end{aligned}$$

They obtained sufficient conditions on initial functions for which there exists a finite time that some solutions blow up at this time.

Tahamtani and Shahrouzi studied the following fourth order viscoelastic equation

$$u_{tt}+\Delta^2u-\int_0^t g(t-s)\Delta^2u(s)\mathrm{d}s=|u|^pu,$$

in a bounded domain and proved the existence of weak solutions in [7]. Furthermore, they showed that there are solutions under some conditions on initial data which blow up in finite time with non-positive initial energy as well as positive initial energy. Later, the same authors investigated global behavior of solutions to some class of inverse source problems. In [8], the same authors investigated the global in time behavior of solutions for an inverse problem of determining a pair of functions $\{u,f\}$ satisfying the equation

$$u_{tt}+\Delta^2u-|u|^pu+a(x,t,u,\nabla u,\Delta u)=f(t)\omega(x), \quad x\in\Omega, \quad t>0,$$

the initial conditions

$$u(x,0)=u_0(x), \quad u_t(x,0)=u_1(x), \quad x\in\Omega,$$