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The Improved Modified Decomposition Method for the Treatment of Systems of Singular Nonlinear Partial Differential Equations with Initial Conditions

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Abstract. New developed inverse differential operators incorporated into the semi-analytical treatment of the modified decomposition method (MDM) are used to solve the systems of first and second-order singular nonlinear partial differential equations (PDEs) with initial conditions arising in physics. The new proposed method is called the improved modified decomposition method (IMDM), and is used to the treatment of a few case study initial-value problems. The results obtained by the IMDM are in full agreement with the existing exact analytical solutions.

AMS Subject Classifications: 35F25, 35G25, 35C10, 35A20, 35Q35

Key Words: Systems of singular nonlinear PDEs; initial conditions; IMDM; inverse operators.

1 Introduction

Systems of singular nonlinear partial differential equations (PDEs) appear in a lot of cases in physics such as cylindrical and spherical Kortweg-de-Vries equation, Cauchy momentum equations, and the reaction-diffusion equations. Nevertheless, these systems can not be easily solved. Various numerical methods have been introduced for the solution of the systems of singular nonlinear PDEs, but they are usually developed for specific problems, and do not have the capability to generally solve these systems.

The development of semi-analytical methods have opened a new horizon on solving various mathematical and physical equations in recent years. These methods give approximate analytical series solutions. Homotopy perturbation method (HPM), Homotopy analysis method (HAM), Adomian decomposition method (ADM), modified decomposition method (MDM) and differential transform method (DTM) are a few of the

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semi-analytical methods. In the semi-analytical Adomian decomposition method (ADM) and modified decomposition method (MDM), an inverse linear differential operator is developed for the equation in hand and then infinite series are substituted for the dependent variable and any nonlinear terms available in the equation. The ADM and the MDM have been applied for solving various mathematical and physical equations. However, they have previously never been used for solving the systems of singular nonlinear PDEs.

Wazwaz [1] applied the decomposition method to systems of PDEs and to reaction-diffusion Brusselator equation. In another paper, Wazwaz [2] applied the MDM to the problem of transient flow of gas through a porous medium. Wazwaz [3] used the MDM for the analytic treatment of variable-coefficint fourth-order parabolic PDEs. Wazwaz [4, 5] presented an ADM algorithm for solving differential equations of Lane-Emden type and a ADM treatment for mixed Volterra-Fredholm integral equations. Wazwaz [6] also used the ADM for solving the Bratu-type equations. Ray [7] applied the MDM for the solution of the coupled Klein-Gordon-Schrodinger equation. Noor et al. [8] used MDM for solving initial and boundary-value problems using Pade approximants. Muttaqi and co-workers [9] applied the MDM for solving the nonlinear Voltera-Fredholm integro-differential equation. Khan and Hussain [10] applied the combined Laplace transformation-decomposition method on the semi-infinite domain. Khan et al. [11] proposed the Auxiliary Laplace Parameter Method (ALPM) using Adomian polynomials and Laplace transformation for solving nonlinear differential equations. In a paper by Mosta and Sibanda [12], a novel numerical approach, based on a new application of the successive linearization method (SLM), was presented for the solution of a class of singular nonlinear boundary-value problems arising in physiology. Khan and Gondal [13] constructed a new mechanism for the solution of Abel's type singular integral equations, i.e. the two-step Laplace decomposition algorithm (TSLDA). Kumar and Singh [14] discussed the physical processes of a thin liquid film evolution over a wet surface down a vertical wall, derived the classical thin film equation, which is a nonlinear third-order singular ordinary differential equation, and then obtained series solution by employing ADM.

In the present study, new inverse developed differential operators incorporated into the MDM are used to solve the first and second-order systems of singular nonlinear PDEs with initial conditions arising in physics. The results of the solutions are compared with the existing exact solutions of the systems of singular nonlinear PDEs, which indicate excellent agreement.

2 Application of IMDM to systems of singular nonlinear PDEs

2.1 Systems of general first-order singular nonlinear PDEs

We consider the system of general first-order (in t) coupled singular nonlinear PDEs as follows: