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A New Blowup Criterion for Ideal Viscoelastic Flow

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Abstract. In this paper, we give a new blowup criteria of strong solutions to the Oldroyd model for ideal viscoelastic flow.

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1 Introduction and main results

In this short note, we will give the Beale-Kato-Majda type criteria for the breakdown of smooth solutions to the incompressible ideal viscoelastic flow of the following system

$$\begin{cases} \partial_t u + u\cdot\nabla u + \nabla p = \nabla\cdot FF^T, \\ \partial_t F + (u\cdot\nabla)F = \nabla u F, \\ \nabla\cdot u = 0, \end{cases} \tag{1.1}$$

where $x\in\mathbb{R}^3, t\geq 0, u$ is the flow velocity, $F=F(x,t)$ represents the local deformation gradient of the fluid. This system of partial differential equations arises in the Oldroyd model for ideal viscoelastic flow i.e. a viscoelastic fluid whose elastic properties dominate its behavior (see [1]). Global existence for solutions near equilibrium of the viscous analog of (1.1) has been verified by Lin et al. [1] and Lei et al. [2]; A further discussion of these topics can be found in the work Lin and Zhang [3]. Noting the second equation of system

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(1.1), if $\nabla F_k = 0$ initially, it will remain so for later times. In what follows, we will make this assumption and also note that it implies the equality

$$\nabla\cdot FF^T = \sum_{k=1}^{3}(F_k\cdot\nabla)F_k.$$

Under the assumed properties of the deformation gradient in above references, the system (1.1) can be rewritten as eqnarray

$$\begin{cases}\partial_t u + u\cdot\nabla u + \nabla p = \kappa\nabla\cdot\tau,\\ \partial_t\tau + u\cdot\nabla\tau - \mu\Delta\tau - b\tau = Q(\nabla u,\tau) + aDu,\\ \nabla\cdot u = 0,\\ t=0:\ \ u=u_0(x),\ \ \tau=\tau_0(x).\end{cases} \tag{1.2}$$

For the vanishing viscosity case, Sideris and Thomases [4] also established the global existence of smooth solutions near the equilibrium using the incompressible limit by imposing a null condition on the elastic stress. Using the standard energy method [5], it is known that for $(u_0,F_{k0})\in H^s$, $s\ge 3$, there exists $T>0$ such that the Cauchy problem (1.1) has a unique smooth solution $(u(t,x),F(t,x))$ on $[0,T]$ satisfying

$$(u,F)\in C([0,T];H^s)\cap C^1([0,T];H^{s-1}). \tag{1.3}$$

Recently, Hu and Hynd [6] get an analog of the Beale-Kato-Majda criterion [7] for singularities of smooth solutions of the system of PDE arising in the Oldroyd model for ideal viscoelastic flow. More precisely, they showed that if the smooth solution (u,F) satisfies the following condition:

$$\int_0^T \|\nabla\times u\|_{L^\infty}\mathrm{d}t<\infty \ \text{ and } \ \int_0^T \|\nabla\times F\|_{L^\infty}\mathrm{d}t<\infty, \tag{1.4}$$

then the solution (u,F) can be extended beyond $t=T$, namely, for some $T<T^\star$,$(u,F)\in C([0,T^\star);H^s(\mathbb{R}^3))\cap C^1([0,T^\star);H^{s-1}(\mathbb{R}^3))$.

More recently, for the following incompressible Euler equations

$$\begin{cases}u_t + u\cdot\nabla u + \nabla p = 0,\\ \nabla\cdot u = 0,\\ t=0:\ u=u_0.\end{cases} \tag{1.5}$$

H. Kozono et al. [8] refined the result of Beale et al. [7] and showed that if the solution u to (1.3) satisfies

$$\int_0^T \|\nabla\times u\|_{\dot{B}^0_{\infty,\infty}}\mathrm{d}t\le\infty, \tag{1.6}$$