JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **28**, No. 1, pp. 47-73

doi: 10.4208/jpde.v28.n1.6
March 2015

Random Attractor for Stochastic Partly Dissipative Systems on Unbounded Domains

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Received 1 December 2014; Accepted 6 March 2015

Abstract. In this paper, we consider the long time behaviors for the partly dissipative stochastic reaction diffusion equations. The existence of a bounded random absorbing set is firstly discussed for the systems and then an estimate on the solution is derived when the time is sufficiently large. Then, we establish the asymptotic compactness of the solution operator by giving uniform a priori estimates on the tails of solutions when time is large enough. In the last, we finish the proof of existence a pullback random attractor in $L^2(R^n)\times L^2(R^n)$. We also prove the upper semicontinuity of random attractors when the intensity of noise approaches zero. The long time behaviors are discussed to explain the corresponding physical phenomenon.

AMS Subject Classifications: 35R60, 37L55, 60H10

Chinese Library Classifications: O193, O231.3

Key Words: Reaction diffusion equation; random dynamical systems; random attractors; asymptotic compactness; Sobolev compact embedding.

1 Introduction

The aim of this work is to research the long time behavior of asymptotically compact random dynamical systems, which can be generated by solutions of the following stochastic partly dissipative reaction diffusion systems with additive white noise on unbounded domains [1],

$$\mathrm{d}u+(-\mu\Delta u+\lambda u+\alpha v)\mathrm{d}t=(h(u)+f(x))\mathrm{d}t+\varepsilon\sum_{j=1}^{m}h_j\mathrm{d}\omega_j, \tag{1.1}$$

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$$\mathrm{d}v+(\delta v-\beta u)\mathrm{d}t=g(x)\mathrm{d}t+\varepsilon\sum_{j=1}^{m}h_j^*\mathrm{d}\omega_j, \tag{1.2}$$

where μ, λ, α, δ, β are positive constants, f, g, h_j and h_j^* are given functions, $h(u)$ is a nonlinear function satisfying certain dissipative condition and $\{\omega_j\}_{j=1}^m$ are independent two-sided real-valued Wiener processes on a probability space which will be explain later. The without additive white noise equations [2] is often used to describe the signal transmission across axon and is a model of FitzHugh-Nagumo equation in neurobiology. The FitzHugh-Nagumo equation is obtained by simplifying the four variables HudgKin-Huxley equation. And the simplified H-H equation are very successful in many ways in describing the behavior of nerve fiber. The mathematical model has become an important branch of nonlinear science. These equations are known as an excitable system, we can refer to the literature [3–5]. However,deterministic models often ignore many small perturbation, the stochastic model can more accurately describe the physical phenomenon. In the past decades, consider the long time behavior of infinite dimensional dynamical system was one of the most important work in mathematical physics [6–8]. One of the important tasks of investigation of dissipative dynamical system is to find conditions for the existence of an random dynamical system [9]. A RDS on the phase space is said to be dissipative if and only if there exists a bounded random absorbing set [10–12]. The long time behavior of solutions from problem (1.1)-(1.2) in a bounded domain has been studied by several authors [1, 13], but little is known for unbounded domains. Existence of random attractor on unbounded domains for stochastic Benjamin-Bona-Mahony equation and Navier-Stokes equation have investigated distinguish in [14] and [15]. Here we prove the existence of such a random attractor [16] for the partly dissipative stochastic reaction diffusion systems (1.1)-(1.2). It is worth mentioning that many researcher are interested in this research area. As everyone knows that the sobolev embedding are no longer compact in the unboundedness of the domain, which form a major difficulty for proving the existence of an attractor [17]. So the asymptotic compactness of solutions cannot be acquired with the standard method. The energy equation approach is employed by some authors in the deterministic case on unboundedness domain [18]. In this paper, we provide uniform estimates on the far-field values of solutions, which can be used to circumvent the difficulty caused by the unboundedness of the domain. Some authors have used this method. The master devote in this essay is to develop the method of using tail estimates to the case of stochastic dissipative systems [9], and prove the existence of a random attractor for the stochastic partly dissipative reaction diffusion systems in $L^2(R^n)\times L^2(R^n)$.

The paper is made as follows. In the Section 2, we recall some main definitions and results concerning the existence of a random attractor for random dynamical systems. In Section 3, we transform (1.1)-(1.2) into a deterministic systems with random parameter and come into being a continuous random dynamical system. In Section 4, we devote to obtaining uniform estimates of solutions when $t\to\infty$. These estimates are necessary