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(G'/G^2)-expansion Solutions to MBBM and OBBM Equations

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Abstract. In this paper, with the aid of symbolic computation, the modified Benjamin-Bona-Mahony and Ostrovsky-Benjamin-Bona-Mahony equations are investigated by extended (G'/G^2)-expansion method. As a consequence, some trigonometric, hyperbolic and rational function solutions with multiple arbitrary parameters for the two equations are revealed, which helps to illustrate the effectiveness of this method.

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1 Introduction

Because of the profound significance of nonlinear evolution equations (NLEEs) arising from physics and applied sciences, researchers have devoted a lot of effort to study them. One of the fundamental problems is to construct as many and general as possible exact solutions of NLEEs. Rather surprisingly, there are a number of methods have been well established. One of these methods is the basic (G'/G)-expansion method [1], which has been widely utilized to seek many exact solutions of NLEEs [2, 3], discrete nonlinear equations [4], and fractional differential equations [5].

Based on the basic (G'/G)-expansion method, many advances have been made in different manners such as the extended (G'/G)-expansion method [6], the generalized (G'/G)-expansion method [7, 8], the modified (G'/G)-expansion method [9], the improved (G'/G)-expansion method [10], the two-variable $(G'/G, 1/G)$-expansion method

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[11], the extended discrete (G'/G)-expansion method [12], the generalized and improved (G'/G)-expansion method [13], the further improved (G'/G)-expansion method [14], the multiple (G'/G)-expansion method [15], the generalized extended (G'/G)-expansion method [16], the (ω/g)-expansion method [17], and so on. These developed methods are used to find traveling wave solutions of many differential equations.

This article is arranged as follows. In Section 2, we outline the main steps of the extended method. In Section 3, we apply the extended method to the modified Benjamin-Bona-Mahony (MBBM) and Ostrovsky-Benjamin-Bona-Mahony (OBBM) equations respectively to find out many exact solutions. Lastly, a brief summary of the results is given.

2 Methodology

Consider a given NLEE with $u=u(t, x, y, \cdots)$ as

$$P(u, u_t, u_x, u_y, \cdots)=0. \tag{2.1}$$

Step 1. Reduction of a NLEE (2.1) into an ODE (2.2). Introducing a variation $u(t, x, y, \cdots)=u(\xi)$, $\xi=sx+ly+\cdots-Vt+d$, where $s, l, \cdots, V, d$ are undetermined constants, we can convert Eq. (2.1) into an ODE

$$O(u, u', u'', u''', \cdots)=0. \tag{2.2}$$

Step 2. Construction of the formal solution of Eq. (2.2). We assume that Eq. (2.2) has the solution in the form

$$u(\xi)=a_0+\sum_{i=1}^{m}\left[a_i\left(\frac{G'}{G^2}\right)^{i}+b_i\left(\frac{G'}{G^2}\right)^{-i}\right], \tag{2.3}$$

where $G=G(\xi)$ satisfies

$$\left(\frac{G'}{G^2}\right)'=\mu+\lambda\left(\frac{G'}{G^2}\right)^2, \tag{2.4}$$

in which $\mu\neq 1$ and $\lambda\neq 0$ are integers, while a_0, a_i, b_i $(i=1, 2, \cdots, m)$ are constants to be determined. From Eq. (2.2), we can determine the positive integer m by homogeneous balance principle.

Step 3. Determination of unknown constants. Substituting (2.3) along with (2.4) into Eq. (2.2), collecting all terms with the same powers of $(G'/G^2)^j$ $(j=0, \pm1, \pm2, \cdots)$, and equating zero of all the coefficients yields a set of algebraic equations. By simple calculation with symbolic computation software, the solutions of the algebraic equations can be derived.