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Doubly Perturbed Neutral Stochastic Functional Equations Driven by Fractional Brownian Motion

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Abstract. In this paper, we study a class of doubly perturbed neutral stochastic functional equations driven by fractional Brownian motion. Under some non-Lipschitz conditions, we will prove the existence and uniqueness of the solution to these equations by providing a semimartingale approximation of a fractional stochastic integration.

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Key Words: Fractional Brownian motion; doubly perturbed neutral functional equations; non-Lipschitz condition.

1 Introduction

As the limit process from a weak polymer model, the following doubly perturbed Brownian motion

$$X_t = B_t + a \max_{0\le s\le t} X_s + b \min_{0\le s\le t} X_s, \tag{1.1}$$

was presented in Norris, Rogers and Williams [1], also as the scaling limit of some self-interacting random walks [2], has attracted much interest from several directions, see Le Gall and Yor [3], Davis [4, 5], Carmona, Petit and Yor [6], Perman and Werner [7], Chaumont and Doney [8, 9], Werner [10], etc.

Following them, Doney and Zhang [11] have studied the following single perturbed stochastic functional equation:

$$X_t = X_0 + \int_0^t b(s,X_s)\mathrm{d}s + \int_0^t \sigma(s,X_s)\mathrm{d}B_s + a \max_{0\le s\le t} X_s, \tag{1.2}$$

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with the condition that σ, b are Lipschitz continuous functions.

Recently, under some non-Lipschitz conditions, Luo [12] obtained the existence and uniqueness of the solution to the following doubly stochastic functional equation

$$X_t = X_0 + \int_0^t b(s,X_s)\mathrm{d}s + \int_0^t \sigma(s,X_s)\mathrm{d}B_s + \int_0^t \int_{-1}^{\infty} h(X_{s-},y)\widetilde{N}(\mathrm{d}s,\mathrm{d}y) + a\max_{0\le s\le t} X_s + b\min_{0\le s\le t} X_s, \tag{1.3}$$

Hu and Ren [13] studied the existence and uniqueness of the solution to the following doubly perturbed neutral stochastic functional equation

$$X_t = X_0 + G(t,X_t) - G(0,X_0) + \int_0^t f(s,X_s)\mathrm{d}s + \int_0^t \sigma(s,X_s)\mathrm{d}B_s + a\max_{0\le s\le t} X_s + b\min_{0\le s\le t} X_s, \tag{1.4}$$

and Liu and Yang [14] proved the existence and uniqueness of the solution to the following doubly perturbed neutral stochastic functional equation with Markovian switching and Poisson jumps

$$X_t = X_0 + G(X_t,r(t)) - G(0,r_0) + \int_0^t f(s,r(s),X_s)\mathrm{d}s + \int_0^t \sigma(s,r(s),X_s)\mathrm{d}B_s + \int_0^t \int_{-\tau}^{\infty} h(X_{s-},y)\widetilde{N}(\mathrm{d}s,\mathrm{d}y) + a\max_{0\le s\le t} X_s + b\min_{0\le s\le t} X_s. \tag{1.5}$$

One solution for many SDEs is a semimartingale as well a Markov process. However, many objects in real world are not always such processes since they have long-range aftereffects. Since the work of Mandelbrot and Van Ness [15], there is an increasing interest in stochastic models based on the fractional Brownian motion. A fractional Brownian motion (fBm) of Hurst parameter $H\in(0,1)$ is a centered Gaussian process $B^H=\{B^H(t),t\ge 0\}$ with the covariance function

$$R_H(t,s) = \mathbb{E}(B_t^H B_s^H) = \frac{1}{2}(t^{2H} + s^{2H} - |t-s|^{2H}).$$

When $H=1/2$ the fBm becomes the standard Brownian motion, and the fBm B^H neither is a semimartingale nor a Markov process if $H\neq 1/2$. However, the fBm B^H, $H>1/2$ is a long-memory process and presents an aggregation behavior. The long-memory property make fBm as a potential candidate to model noise in mathematical finance (see [16]); in biology (see [17]); in communication networks (see, e.g., [18]); the analysis of global temperature anomaly [19] electricity markets [20] etc.

In [15], Mandelbrot et al. have given a representation of B_t^H of the form:

$$B_t^H = \frac{1}{\Gamma(1+\alpha)}\Big(U(t) + \int_0^t (t-s)^{\alpha}\mathrm{d}W_s\Big),$$