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Exact Traveling Wave Solutions for Higher Order Nonlinear Schrödinger Equations in Optics by Using the $(\frac{G'}{G},\frac{1}{G})$-expansion Method

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Abstract. The propagation of the optical solitons is usually governed by the nonlinear Schrödinger equations. In this article, the two variable $(\frac{G'}{G},\frac{1}{G})$-expansion method is employed to construct exact traveling wave solutions with parameters of two higher order nonlinear Schrödinger equations describing the propagation of femtosecond pulses in nonlinear optical fibers. When the parameters are replaced by special values, the well-known solitary wave solutions of these equations rediscovered from the traveling waves. This method can be thought of as the generalization of well-known original $(\frac{G'}{G})$-expansion method proposed by M. Wang et al. It is shown that the two variable $(\frac{G'}{G},\frac{1}{G})$-expansion method provides a more powerful mathematical tool for solving many other nonlinear PDEs in mathematical physics.

AMS Subject Classifications: 35Q51, 37K10, 35K99, 35P05

Key Words: The two variable $(\frac{G'}{G},\frac{1}{G})$-expansion method; Schrödinger equations; exact traveling wave solutions; Solitary wave solutions.

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1 Introduction

During the past decades, investigations on the optical fiber communications have become more and more attractive [1], in which the optical solitons have their potential applications in optical fiber transmission systems [2, 3]. As we all know, solitonic structures are seen in many fields of sciences and engineering [4, 5], among which an optical soliton exists in a fiber on the basis of the exact balance between the group velocity dispersion

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(GVD) and the self-phase modulation (SPM). The propagation of the optical solitons is usually governed by the nonlinear Schrödinger equation (NLSE), which is one of the most important models in modern nonlinear science. Moreover, much attention has been paid to the investigation on the generalized NLSEs with constant coefficients as a kind of ideal models of the much more complicated physical problems [6, 7]. As a matter of fact, in a real fiber there exist some fiber nonuniformities to influence various effects such as the gain or loss, GVD and SPM, etc.

In this paper, we consider the following two higher order nonlinear Schrödinger equations:

The first one is the higher order nonlinear Schrödinger equation [8]:

$$q_z = i\alpha_1 q_{tt} + i\alpha_2 q|q|^2 + \alpha_3 q_{ttt} + \alpha_4\left(q|q|^2\right)_t + \alpha_5 q\left(|q|^2\right)_t, \tag{1.1}$$

which describes the propagation of ultrashort femtosecond pulses in nonlinear optical fibers, where the complex function $q = q(z,t)$ is slowly varying envelop of the electric field, the subscripts z and t are the spatial and temporal partial derivative in retard time coordinates, and α_1, α_2, α_3,α_4 and α_5 are real nonzero parameters related to the group velocity, self-phase modulation, third-order dispersion, selfsteepening, and self-frequency shift arising from stimulated Raman scattering, respectively. Eq. (1.1) has been discussed using the $(\frac{G'}{G})$-expansion method and its exact solutions have been found in [8].

The second is the higher order nonlinear Schrödinger equation [9]:

$$iq_z + q_{tt} + 2q|q|^2 + i\alpha_1 q_{ttt} + + i\alpha_2\left(q|q|^2\right)_t + i\alpha_3\left(q|q|^4\right)_t + \alpha_4 q|q|^4 = 0, \tag{1.2}$$

which describes the propagation of femtosecond pulses in nonlinear optical fibers, where the complex function $q = q(z,t)$ is a complex function that represents a normalized complex amplitude of the pulse envelope in nonlinear optical fiber, the variable z is interpreted as the normalized propagation distance, t-retarded time, and α_1, α_2, α_3 and α_4 are real nonzero constants. Eq. (1.2) has been discussed in [9] using a sub-ODE method and its exact solutions have been found.

Exact solutions (especially traveling wave solutions) of nonlinear PDEs play an important role in the study of nonlinear physical phenomena. Many powerful methods have been presented, such as the inverse scattering method [10], the Hirota bilinear transform method [11], the truncated Painlevé expansion method [12,13], the Bäcklund transform method [14], the exp-function method [15,16], the tanh-function method [17-19], the Jacobi elliptic function expansion method [20,21], the $(\frac{G'}{G})-$expansion method [22-26], the modified $(\frac{G'}{G})$-expansion method [27,28], the $(\frac{G'}{G},\frac{1}{G})$-expansion method [29-37], and so on. The key idea of the one variable $(\frac{G'}{G})$-expansion method is that the exact solutions of nonlniear PDEs can be expressed by a polynomial in one variable $(\frac{G'}{G})$ in which $G=G(\xi)$ satisfies the second order linear ODE $G''(\xi)+\lambda G'(\xi)+\mu G(\xi)=0$, where λ and μ are constants and $' = \frac{d}{d\xi}$. The key idea of the two variable $(\frac{G'}{G},\frac{1}{G})$-expansion method is that