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Solitary Wave Solutions to the ZKBBM Equation and the KPBBM Equation via the Modified Simple Equation Method

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Abstract. In this article, the modified simple equation method (MSE) is used to acquire exact solutions to nonlinear evolution equations (NLEEs) namely the Zakharov-Kuznetsov Benjamin-Bona-Mahony equation and the Kadomtsov-Petviashvilli Benjamin-Bona-Mahony equation which have widespread usage in modern science. The MSE method is ascending and useful mathematical tool for constructing exact traveling wave solutions to NLEEs in the field of science and engineering. By means of this method we attained some significant solutions with free parameters and for special values of these parameters, we found some soliton solutions derived from the exact solutions. The solutions obtained in this article have been shown graphically and also discussed physically.

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1 Introduction

During the last few decades there has been considerable research on water wave equations and the stability of their solitary waves which are all governed by nonlinear evolution equations (NLEEs). NLEEs is one of the fundamental area of applied analysis,

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such as, fluid mechanics, quantum mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics etc. It is so much difficult to imagine any area of applications where its influence is not felt. One of the special analytical solutions called traveling wave solutions of NLEEs lead a great importance as the largest part of the complex physical phenomena arising in applied science and engineering fields can be described by NLEEs. Therefore, investigating exact traveling wave solutions to NLEEs by using several methods has attracted lots of attention from numerous researchers who are concern with nonlinear science. NLEEs are more difficult to examine and it is not easy to achieve their solitary solutions. In order to examine their solitary solutions diverge group of mathematicians, physicists and engineers are working simultaneously. Thus, several techniques for finding exact solutions to NLEEs have been established. Some of these are: the homotopy perturbation method [1-3], the Jacobi elliptic function method [4], the truncated Painleve expansion method [5], the He's semi-inverse variational principle [6-7], the variational method [8-11], the Backlund transformation [12], the Miura transformation [13] method, the F-expansion method [14], the generalized Riccati equation [15], the homogenous balance method [16-17], the trial function method [18], the $\exp(-\varphi(\xi))$-expansion method [19-21], the tanh-function method [22-24], the inverse scattering method [25], the sine-cosine method [26-27], the Exp-function method [28-30], the asymptotic method [31], the Hirota's bilinear transformation method [32-33], the auxiliary equation method [34], the non-perturbative method [35], the (G'/G)-expansion method [36-43], the improved (G'/G)expansion method [44], the soliton perturbation theory [45-47], etc.

In the previous literature many authors studied the ZKBBM equation and KPBBM equation, such as, Das et al [48] derived the solution of ZKBBM equation by using variation of (G'/G)-expansion method and Martnez et al [49] used Feng's first integral method for finding exact solutions of ZKBBM equation. Kumar et al [50] solved KPBBM equation by means of tan-cot method and Alam et al [51] derived the solutions of KPBBM equation by using generalized (G'/G)-expansion method. The above study shows that several methods have been adopted to achieve exact solutions to the above equations.

But no one has studied these equations by making use of the MSE method. The objective of this article is that, we will make use of the MSE method to extract new exact traveling wave solutions and then solitary wave solutions to these equations.

The article is organized as follows: In Section 2, we describe the MSE method. In Section 3, we apply the method into two nonlinear evolution equations mentioned above. In Section 4, we give the physical interpretation and graphical representation of the obtained solutions. In Section 5, discussions are presented and finally in section 6, we have drawn our conclusions.