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Decay of a Transmission Problem with Memory and Time-Varying Delay

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Abstract. In this article we consider a transmission problem with memory in a bounded domain and varying delay term in the first equation. Under suitable assumptions on the weight of the damping and the weight of the delay, we show the exponential stability of the solution by introducing a suitable Lyapunov functional.

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Key Words: Wave equation; transmission problem; varying delay term; memory; exponential stability.

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1 Introduction

In this article, we consider the transmission problem with a varying delay term,

$$\begin{cases} u_{tt}(x,t)-au_{xx}(x,t)+\int_0^t g(t-s)u_{xx}(x,s)\mathrm{d}s+\mu_1 u_t(x,t) \\ \qquad +|\mu_2|u_t(x,t-\tau(t))=0, \quad (x,t)\in\Omega\times(0,+\infty), \\ v_{tt}(x,t)-bv_{xx}(x,t)=0, \qquad\quad (x,t)\in(L_1,L_2)\times(0,+\infty), \end{cases} \tag{1.1}$$

where $0<L_1<L_2<L_3$, $\Omega=]0,L_1[\cup]L_2,L_3[$, a,b,μ_1, are positive constants, and μ_2 is a real number $\tau(t)>0$ is the delay function.

We assume that there exist positive constants $\tau_0,\overline{\tau}$ such that

$$0<\tau_0\leq\tau(t)\leq\overline{\tau}. \tag{1.2}$$

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System (1.1) is subjected to the following boundary and transmission conditions:

$$\begin{cases} u(0,t)=u(L_3,t)=0, & \\ u(L_i,t)=v(L_i,t), & i=1,2, \\ \left(a-\int_0^t g(s)\mathrm{d}s\right)u_x(L_i,t)=bv_x(L_i,t), & i=1,2, \end{cases} \tag{1.3}$$

and the initial conditions:

$$\begin{cases} u(x,0)=u_0(x), \quad u_t(x,0)=u_1(x), & x\in\Omega, \\ u(x,t-\tau(t))=f_0(x,t-\tau(t)), & x\in\Omega, t\in[0,\overline{\tau}], \\ v(x,0)=v_0(x), \quad v_t(x,0)=v_1(x), & x\in]L_1,L_2[. \end{cases} \tag{1.4}$$

Moreover, we assume that

$$\tau\in W^{2,\infty}([0,T]), \quad \forall T>0, \tag{1.5}$$

$$\tau'(t)\leq d<1, \quad \forall t>0. \tag{1.6}$$

The problem (1.1)-(1.4) related to the wave propagation over a body are called transmission problems, which consists of two different materials, the elastic part and the viscoelastic part. In recent years, many authors have investigated wave equations with viscoelastic damping and showed that the dissipation produced by the viscoelastic part can produce the decay of the solution see [5-7] and the references therein.

For example, Calvalcanti et al. in [8] studied the following equation,

$$u_{tt}-\Delta u+\int_0^t g(t-\tau)\Delta u(\tau)\mathrm{d}\tau+a(x)u_t+|u|^\gamma u=0, \quad \text{in } \Omega\times(0,\infty),$$

where $a:\Omega\to\mathbb{R}_+$ under the conditions that $a(x)\geq a_0>0$ on $w\subset\Omega$, with w satisfying some geometry restrisctions and

$$-\zeta_1 g(t)\leq g'(t)\leq -\zeta_2 g(t), \quad t\geq 0, \tag{1.7}$$

the authors showed the exponential decay. Then Berrimi and Messaoudi [5] obtained the same result under weaker conditions proved on both a and g. Kirane and Said-Houari [9] considered the viscoelastic wave equation with delay

$$u_{tt}-\Delta u+\int_0^t g(t-s)\Delta u(,x-s)\mathrm{d}s+\mu_1 u_t(x,t)+\mu_2 u_t(x,t-\tau)=0, \quad \text{in } \Omega\times(0,\infty),$$

where μ_1, and μ_2 are positive constants. They established a general energy decay result under the condition that $0\leq\mu_2\leq\mu_1$. Later , Liu, [10] improved this result by considering the equation with time-varying delay term, with not necessarly positive coefficient μ_2 of the delay tarm. For $\mu_2=0$, system (1.1)-(1.4) has been investigated in [4]; for $\Omega=[0,L_1]$, the authors showed the well-posedness and exponential stability of the total energy. Muñoz