JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **29**, No. 4, pp. 269-285

doi: 10.4208/jpde.v29.n4.2
December 2016

On Local Existence and Blow-up of a Moving Boundary Problem in 1-D Chemotaxis Model

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Received 3 March 2016; Accepted 3 September 2016

Abstract. In this paper, the local existence and uniqueness of a chemotaxis model with a moving boundary are considered by the contraction mapping principle, and the explicit expression for the moving boundary is formulated. In addition, the finite-time blowup and chemotactic collapse of the solution for such kind of problem are discussed.

AMS Subject Classifications: 35A01, 35K57, 35R35, 47D03

Chinese Library Classifications: O175.29

Key Words: Chemotaxis model; moving boundary; local existence; finite-time blowup.

1 Introduction

In this paper, we consider the following moving boundary problem of a chemotaxis model:

$$\begin{cases} u_t=\nabla(\nabla u-\chi u\nabla v)+u, & \text{in } \Omega_t\times(0,T),\\ u=0, & \text{in } \Omega\times(0,T)\setminus\Omega_t\times(0,T),\\ -\nabla u\cdot\dfrac{\nabla\Phi}{|\nabla\Phi|}=k(x,t)u, & \text{on } \Gamma_t=\partial\Omega_t\times(0,T),\\ u\dfrac{\partial\Phi}{\partial t}=\nabla u\cdot\nabla\Phi-\chi u\nabla v\cdot\nabla\Phi, & \text{on } \Gamma_t=\partial\Omega_t\times(0,T),\\ u(x,0)=u_0(x), & \text{in } \Omega_0,\\ 0=\Delta v+u-e^t, & \text{in } \Omega\times(0,T),\\ \dfrac{\partial v}{\partial n}=0, & \text{on } \partial\Omega\times(0,T), \end{cases} \tag{1.1}$$

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where

- $u=u(x,t)$ is an unknown function of $(x,t)\in\Omega_t\times(0,T)$ and it stands respectively for the density of the considered species;
- $v=v(x,t)$ is an unknown function of $(x,t)\in\Omega\times(0,T)$ and it stands that the chemical which triggers the movement;
- χ, γ, μ and β are positive constants parameters;
- Ω is a bounded open subset in $R^N(N\geq 1)$ with smooth boundary $\partial\Omega$;
- n is unit outer normal vector of $\partial\Omega$;
- $\Gamma_t:\ \Phi(x,t)=0$ is an unknown moving boundary.

The original model which was introduced by Keller and Segel [1] reads as follows

$$\begin{cases} u_t=\Delta u-\chi\nabla(u\nabla v), & x\in\Omega, t>0,\\ v_t=\gamma\Delta v-\mu v+\beta u, & x\in\Omega, t>0,\\ u(x,0)=u_0,\ v(x,0)=v_0, & u_0,v_0\geq 0, x\in\Omega,\\ \dfrac{\partial u}{\partial n}=\dfrac{\partial v}{\partial n}=0, & x\in\partial\Omega, t>0. \end{cases} \tag{1.2}$$

The problem (1.2) is intensively studied by many authors and most results have been devoted to the investigation of some limit cases corresponding to particular choices of the parameters χ, γ, μ and β above. One of them is that the diffusive velocity of γ tends to infinity, which leads to the following system (see [2])

$$\begin{cases} u_t=\Delta u-\chi\nabla(u\nabla v), & x\in\Omega, t>0,\\ 0=\Delta v+(u-1), & x\in\Omega, t>0,\\ \dfrac{\partial u}{\partial n}=\dfrac{\partial v}{\partial n}=0, & x\in\partial\Omega, t>0,\\ u(x,0)=u_0,\ u_0\geq 0, & x\in\Omega, \end{cases} \tag{1.3}$$

where $\int_\Omega u_0 \mathrm{d}x=|\Omega|,|\Omega|$ is the volume of Ω. For the problem (1.3), many results have been gained by some authors (see, e.g., [2–17]).

Since the spatial diffusive velocity of v is much faster than that of u, it makes sense that the spatial domain occupied by u is a subset of the spatial domain occupied by v at the same time. In other words, let $\Omega\subset R^N$ be a bounded open domain and $\Omega_0\subset\subset\Omega$ be an open sub-domain. Assume a population density $u(x,0)$ occupying the domain Ω_0, and in the outside of Ω_0 the population density $u(x,0)\equiv 0$ and the external signal v occupying