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VACUUM STATES AND EQUIDISTRIBUTION OF THE RANDOM SEQUENCE FOR GLIMM'S SCHEME (CONTINUATION) ①

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Abstract

This is a continuation of paper [1]. The difference between this paper and paper [1] is that the initial functions considered here are step functions and those considered in [1] are Lipschitz continuous. Since there are centered rarefaction waves here, more delicate techniques are needed. It may be a necessary step in solving p-System with general initial functions by Glimm's scheme. Notice that this paper can not be deduced from [1].

Consider the initial value problem for isentropic gas dynamics in Lagrangian coordinates, so call p-System,

$$v_t - u_x = 0, \quad u_t + p(v)_x = 0, \quad (0, \infty) \times (-\infty, \infty) \tag{P}$$

$$(v(0, x), u(0, x)) = (v_0(x), u_0(x)), \quad (-\infty, \infty) \tag{I}$$

where the pressure $p = p(v) > 0$ is a C^2 function of the specific volume $v > 0$ and u is the velocity of the gas. We assume that $p'(v) < 0$, $p''(v) > 0$ and $\int_1^\infty \sqrt{-p'(v)}\, dv < \infty$. The Riemann invariants are taken as

$$r(u, v) = u + \Phi(v), \quad s(u, v) = u - \Phi(v), \quad \Phi(v) = \int_1^v \sqrt{-p'(s)}\, ds$$

Theorem *If* $u_0(x)$ *and* $v_0(x)$ *are bounded step functions,* $0 < V_* \leq v_0(x) \leq V^* < \infty$, *satisfying conditions*

$$|\Phi(v_0(x_2)) - \Phi(v_0(x_1))| < u_0(x_2) - u_0(x_1), \quad x_1 < x_2 \tag{M_1}$$

i. e.

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$$r_0(x_1) \leq r_0(x_2), \qquad s_0(x_1) \leq s_0(x_2), \qquad x_1 < x_2 \qquad (M_2)$$

and

$$r_0(x+0) - s_0(x-0) < 2\Phi(\infty), \qquad -\infty < x < \infty, \qquad (V)$$

where $r_0(x) = r(u_0(x), v_0(x))$, $s_0(x) = s(u_0(x), v_0(x))$. *Suppose that the random sequence* $a \equiv \{a_n\}$ *is uniformly equidistributed on the interval* $(-1, 1)$. *For given* $T > O$, *if the mesh lengths* $l > 0$, $h > 0$ *are sufficiently small, the ratio* $\delta \equiv lh^{-1} > \lambda_*$, *where* $\lambda_* \equiv \sqrt{-p'(V_*)}$, λ *is a constant, then the Glimm's approximations* $(u_h(t, x), v_h(t, x))$ *of* (P), (I) *are uniformly bounded with respect to* h *in the zone* $(0, T) \times (-\infty, \infty)$.

Condition (V) assures that there is no vacuum at the initial instant.

We refine the definition of uniformly equidistributed sequence given in [1].

Definition A sequence $a \equiv \{a_n\}$ is uniformly equidistributed on the interval $(-1, 1)$, if there is a constant e, $0 < e < \frac{1}{3}$, and a constant $D = D(\varepsilon) > 0$, such that

$$|B(j, n, I) - 2^{-1} n\mu(I)| < Dn^{\varepsilon} \qquad (D)$$

$n = 1, 2, \ldots$holds for any integer $j \geq 1$ and any subinterval I in the interval $(-1, 1)$, where $B(j, n, I)$ denotes the number of m, $j \leq m \leq j+n-1$, with $a_m \in I$, and $\mu(I)$ is the length of I. The constant $D(\varepsilon) > 0$ is independent of j and I.

Uniformly equidistributed sequence can easily be constructed.

Before proving the theorem, we give the following lemmas. Set $f_k^n = f_k(nh, kl)$, here $f = u, v, r, s$, etc., $n + k =$even.

Lemma 1 *For given integers* $n \geq 0$, $q > 0$ *and constant* b, *if*

$$0 \leq r_{k+2}^n - r_k^n, \qquad 0 \leq s_{k+2}^n - s_k^n,$$

$$r_{k+2q}^n - r_k^n \leq b, \qquad s_{k+2q}^n - s_k^n \leq b$$

hold for every k, then

$$0 \leq r_{k+1}^{n+1} - r_{k-1}^{n+1}, \qquad 0 \leq s_{k+1}^{n+1} - s_{k-1}^{n+1},$$

$$r_{k+2q-1}^{n+1} - r_{k-1}^{n+1} \leq b, \qquad s_{k+2q-1}^{n+1} - s_{k-1}^{n+1} \leq b$$

The lemma in paper [1] is a special case of above Lemma 1 as $q = 1$. The proofs of the two lemmas are similar.

The following lemma is trivial.

Lemma 2 *For given* $0 < \underline{V} < \overline{V} < \infty$, *there are constants* $0 < c_* < c^* < \infty$, *such that*

$$c_*(\Phi(v_2) - \Phi(v_1)) < \Phi'(v_1) - \Phi'(v_2) \leq c^*(\Phi(v_2) - \Phi(v_1))$$

hold for all v_1, v_2 $0 < \underline{V} \leq v_1 < v_2 \leq \overline{V} < \infty$.

According to condition (M), the Glimm's approximations under consideration consist of rarefaction waves. Hereafter rarefaction waves are simply called waves. If wave γ is issued from point (nh, kl), $n + k =$odd, the (nh, kl) is the starting point of γ, denoted by $P(\gamma) = (n, k)$. The maximum (minimum) value of wave γ is defined by